

$$\Rightarrow \frac{dy}{dx} = \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$$

$$\frac{dy}{dx} = \frac{4\cos\theta - \cos 2\theta}{-4\sin\theta}$$

$$\frac{dy}{dx} = \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$$

Section B

7) i)  $x = \cos\theta \quad y = \sin\theta - \frac{1}{8}\sin 2\theta$

At A,  $x = 1$

$$\Rightarrow \cos\theta = 1$$

$$\Rightarrow \theta = 0$$

At B,  $x = -1$

$$\Rightarrow \cos\theta = -1$$

$$\Rightarrow \theta = \pi$$

At C,  $x = 0$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

when  $\theta = \frac{\pi}{2}$ ,  $y = 1 - 0 = 1$

when  $\theta = \frac{3\pi}{2}$ ,  $y = -1 - 0 = -1$

$\therefore C$  is point  $(0, 1)$

7) ii)

$$\frac{dy}{d\theta} = \cos\theta - \frac{1}{4}\cos 2\theta$$

$$\frac{dx}{d\theta} = -\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

At max pt  $\frac{dy}{dx} = 0$

$$\Rightarrow \cos 2\theta - 4\cos\theta = 0$$

$$\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$$

7) iii)  $2\cos^2\theta - 4\cos\theta - 1 = 0$

$$\cos\theta = \frac{4 \pm \sqrt{16 + 8}}{4}$$

$$\cos\theta = 2.225 \text{ or } -0.2247$$

$$\Rightarrow \theta = 1.7975 \text{ radians}$$

$$\Rightarrow y = \sin(1.7975) - \frac{1}{8}\sin(2 \times 1.7975)$$

$$y = 1.029 \quad \text{to 4 s.f.}$$

7) iv)

$$y = \frac{1}{4}(4-x)\sqrt{1-x^2}$$

$$\text{Volume} = \int_{-1}^1 \pi y^2 dx$$

$$= \pi \int_{-1}^1 \frac{1}{16} (4-x)^2 (1-x^2) dx$$

$$= \frac{\pi}{16} \int_{-1}^1 (16-8x+x^2)(1-x^2) dx$$

7iv)  

$$\int_{-1}^1 (16 - 8x + x^2 - 16x^2 + 8x^3 - x^4) dx$$
  

$$= \int_{-1}^1 (16 - 8x - 15x^2 + 8x^3 - x^4) dx$$
  

$$= \frac{\pi}{16} \left[ 16x - 4x^2 - 5x^3 + 2x^4 - \frac{x^5}{5} \right]_{-1}^1$$
  

$$= \frac{\pi}{16} \left[ (16 - 4 - 5 + 2 - \frac{1}{5}) - (-16 - 4 + 5 + 2 + \frac{1}{5}) \right]$$
  

$$= \frac{\pi}{16} \left[ 32 - 10 - \frac{2}{5} \right]$$
  

$$= \frac{27\pi}{20} = 4.24$$

$$\angle ABC = 148.2^\circ$$

8iii)  $\vec{r} = \begin{pmatrix} 40 \\ 0 \\ -20 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$

At C

$$\begin{pmatrix} a \\ b \\ 0 \end{pmatrix} = \begin{pmatrix} 40 + 3\lambda \\ 4\lambda \\ -20 + \lambda \end{pmatrix}$$

$$\Rightarrow -20 + \lambda = 0 \Rightarrow \lambda = 20$$

$$\therefore a = 40 + 3 \times 20 = 100$$

$$b = 4 \times 20 = 80$$

8iv)  $\begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} -40 \\ -40 \\ 20 \end{pmatrix} = -240 + 200 + 40 = 0$

$$\begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = 18 - 20 + 2 = 0$$

Since  $\begin{pmatrix} 6 \\ -5 \\ 2 \end{pmatrix}$  is  $\perp$  to two non-parallel lines in plane it is  $\perp$  to plane.

Plane of form

$$6x - 5y + 2z = d$$

A(0, -40, 0) on plane so

$$6(0) - 5(-40) + 2(0) = d$$

$$200 = d$$

Plane is

$$6x - 5y + 2z = 200$$

8i) A(0, -40, 0)  
 B(40, 0, -20)

$$|AB| = \sqrt{(40-0)^2 + (0+40)^2 + (-20-0)^2}$$

$$|AB| = 60 \text{ m}$$

8ii)  $\cos \angle ABC = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}| |\vec{BC}|}$

$$\cos \angle ABC = \frac{\begin{pmatrix} -40 \\ -40 \\ 20 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}}{60 \times \sqrt{3^2 + 4^2 + 1^2}}$$

$$\cos \angle ABC = \frac{-120 - 160 + 20}{60 \sqrt{26}}$$