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8)

i) $P(0, 10, 30)$

$Q(0, 20, 15)$

$R(-15, 20, 30)$

$$\overrightarrow{PQ} = \begin{pmatrix} 0-0 \\ 20-10 \\ 15-30 \end{pmatrix} = \begin{pmatrix} 0 \\ 10 \\ -15 \end{pmatrix}$$

$$\overrightarrow{PR} = \begin{pmatrix} -15-0 \\ 20-10 \\ 30-30 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 0 \end{pmatrix}$$

8 ii) $\begin{pmatrix}2\\3\\2\end{pmatrix}.\overrightarrow{PQ} = \begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}0\\10\\-15\end{pmatrix}$

$= 0 + 30 - 30 = 0$

$\therefore \begin{pmatrix}2\\3\\2\end{pmatrix}$ is $\perp$ to $\overrightarrow{PQ}$

$\begin{pmatrix}2\\3\\2\end{pmatrix}.\overrightarrow{PR} = \begin{pmatrix}2\\3\\2\end{pmatrix}.\begin{pmatrix}-15\\10\\0\end{pmatrix}$

$= -30 + 30 + 0 = 0$

$\therefore \begin{pmatrix}2\\3\\2\end{pmatrix}$ is $\perp$ to $\overrightarrow{PR}$

Since $\begin{pmatrix}2\\3\\2\end{pmatrix}$ is $\perp$ to 2 non-parallel lines in the plane PQR, it is a normal to the plane.

Plane is of form

$2x + 3y + 2z = d$

$P(0, 10, 30)$ on plane so

$2\times 0 + 3\times 10 + 2\times 30 = d$

$\Rightarrow d = 90$

Plane PQR is given by

$2x + 3y + 2z = 90$

iii) $S\left(\frac{0-15}{2}, \frac{20+20}{2}, \frac{15+30}{2}\right)$

$S\left(-\frac{15}{2}, 20, \frac{45}{2}\right)$

$\overrightarrow{PS} = \begin{pmatrix}-\frac{15}{2}\\10\\-\frac{15}{2}\end{pmatrix}$

$\overrightarrow{OT} = \overrightarrow{OP} + \frac{2}{3}\overrightarrow{PS}$

$\overrightarrow{OT} = \begin{pmatrix}0\\10\\30\end{pmatrix} + \frac{2}{3}\begin{pmatrix}-\frac{15}{2}\\10\\-\frac{15}{2}\end{pmatrix}$

$\overrightarrow{OT} = \begin{pmatrix}-5\\16\frac{2}{3}\\25\end{pmatrix}$

$\therefore$ T is point $\left(-5, 16\frac{2}{3}, 25\right)$

iv) Line of drill hole

$\underline{r} = \overrightarrow{OT} + \lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$

$\underline{r} = \begin{pmatrix}-5\\50/3\\25\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\2\end{pmatrix}$

$C(-30, 0, 0)$

If C on drill hole line

$$\left.\begin{aligned} -5 + 2\lambda &= -30 \\ 50/3 + 3\lambda &= 0 \\ 25 + 2\lambda &= 0 \end{aligned}\right\} \quad \begin{aligned} &① \\ &② \\ &③ \end{aligned}$$

From ① $\lambda = -\frac{25}{2}$

From ② $\lambda = -\frac{50}{9}$

From ③ $\lambda = -\frac{25}{2}$

No common value of λ $\therefore$ line does not pass thro C

3) $A(-2, 4, 1)$

$B(2, 3, 4)$

$C(4, 8, 3)$

$\overrightarrow{BA} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$

$\overrightarrow{BA} . \overrightarrow{BC} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} . \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$

$= -8 + 5 + 3 = 0$

$\therefore \overrightarrow{BA}$ and $\overrightarrow{BC}$ are $\perp$

$\angle ABC = 90°$

Area of $\triangle ABC$ =

$\frac{1}{2} \times |BA| \times |BC|$

$= \frac{1}{2} \times \sqrt{16+1+9} \sqrt{4+25+1}$

$= \frac{1}{2} \times \sqrt{26} \sqrt{30}$

≈ 13.964 units^2

5)

i) Point $(2, -1, 4)$

$$\underline{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Plane of form

$$x - y + 2z = c$$

Subst point

$$2 - (-1) + 2(4) = c$$

$$2 + 1 + 8 = c$$

$$11 = c$$

Plane is

$$x - y + 2z = 11$$

ii)

$$\underline{r} = \begin{pmatrix} 7 \\ 12 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$$

Subst in plane

$$7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$$

$$7 + \lambda - 12 - 3\lambda + 18 + 4\lambda = 11$$

$$13 + 2\lambda = 11$$

$$2\lambda = -2$$

$$\lambda = -1$$

Point of intersection is

$$(6, 9, 7)$$

7i) $A(0,0,6)$ $E(15,-20,6)$

$|AE| = \sqrt{15^2+(-20)^2+(6-6)^2}$

$|AE| = 25\text{ m}$

7ii)

Direction BD same as direction AE

$$\vec{AE} = \begin{pmatrix}15\\-20\\0\end{pmatrix} = 5\begin{pmatrix}3\\-4\\0\end{pmatrix}$$

Vector eqn of BD

$$\underline{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + \lambda\begin{pmatrix}3\\-4\\0\end{pmatrix}$$

$$|BD| = 15 \Rightarrow \left|\lambda\begin{pmatrix}3\\-4\\0\end{pmatrix}\right| = 15$$

$$\lambda\sqrt{3^2+(-4)^2+0^2} = 15$$

$$5\lambda = 15$$

$$\lambda = 3$$

D is given by

$$\underline{r} = \begin{pmatrix}-1\\-7\\11\end{pmatrix} + 3\begin{pmatrix}3\\-4\\0\end{pmatrix} = \begin{pmatrix}8\\-19\\11\end{pmatrix}$$

$$D(8,-19,11)$$

7iii)

$A(0,0,6)$
$B(-1,-7,11)$
$C(-8,-6,6)$

$-3(0)+4(0)+5(6) = 30$ ✓

$-3(-1)+4(-7)+5(11)$
$= 3 - 28 + 55 = 30$ ✓

$-3(-8)+4(-6)+5(6)$
$= 24 - 24 + 30 = 30$ ✓

A, B, C on plane $-3x+4y+5z=30$

A Normal is $\begin{pmatrix}-3\\4\\5\end{pmatrix}$

iv) $\vec{BD} = 3\begin{pmatrix}3\\-4\\0\end{pmatrix}$

$$\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}3\\-4\\0\end{pmatrix} = 12-12+0 = 0$$

$\therefore \perp$

$$\vec{AB} = \begin{pmatrix}-1\\-7\\5\end{pmatrix}$$

$$\begin{pmatrix}4\\3\\5\end{pmatrix}.\begin{pmatrix}-1\\-7\\5\end{pmatrix} = -4-21+25 = 0$$

$\therefore \perp$

Since $\begin{pmatrix}4\\3\\5\end{pmatrix}$ is $\perp$ to two non-parallel vectors within plane ABDE, it is normal to the plane.

Plane of form $4x+3y+5z=d$

A on plane so $4(0)+3(0)+5\times6=d$

$\Rightarrow d=30$

Plane is $4x+3y+5z=30$

7v) Angle between planes is same as angle between normals to the planes

$$\cos\theta = \frac{\begin{pmatrix}-3\\4\\5\end{pmatrix}\cdot\begin{pmatrix}4\\3\\5\end{pmatrix}}{\left|\begin{pmatrix}-3\\4\\5\end{pmatrix}\right|\left|\begin{pmatrix}4\\3\\5\end{pmatrix}\right|}$$

$$\cos\theta = \frac{-12+12+25}{\sqrt{(-3)^2+4^2+5^2}\sqrt{4^2+3^2+5^2}}$$

$$\cos\theta = \frac{25}{\sqrt{50}\sqrt{50}}$$

$$\cos\theta = \frac{25}{50} = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

#

8) i)

$A(0, -40, 0)$

$B(40, 0, -20)$

$$|AB| = \sqrt{(40-0)^2 + (0--40)^2 + (-20-0)^2}$$

$$|AB| = 60\text{ m}$$

8ii)

$$\cos \angle ABC = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|}$$

$$\cos A\hat{B}C = \frac{\begin{pmatrix}-40\\-40\\20\end{pmatrix} \cdot \begin{pmatrix}3\\4\\1\end{pmatrix}}{60 \times \sqrt{3^2+4^2+1^2}}$$

$$\cos A\hat{B}C = \frac{-120-160+20}{60\sqrt{26}}$$

$$\angle ABC = 148.2^\circ$$

8 iii)

$$\underline{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$$

At C

$$\begin{pmatrix}a\\b\\0\end{pmatrix} = \begin{pmatrix}40+3\lambda\\4\lambda\\-20+\lambda\end{pmatrix}$$

$$\Rightarrow -20+\lambda = 0 \quad \Rightarrow \lambda = 20$$

$$\therefore \quad a = 40 + 3\times 20 = 100$$

$$b = 4\times 20 = 80$$

8iv)

$$\begin{pmatrix}6\\-5\\2\end{pmatrix} \cdot \begin{pmatrix}-40\\-40\\20\end{pmatrix} = -240+200+40 = 0$$

$$\begin{pmatrix}6\\-5\\2\end{pmatrix} \cdot \begin{pmatrix}3\\4\\1\end{pmatrix} = 18-20+2 = 0$$

Since $\begin{pmatrix}6\\-5\\2\end{pmatrix}$ is $\perp$ to two non-parallel lines in plane it is $\perp$ to plane.

Plane of form

$$6x - 5y + 2z = d$$

$A(0, -40, 0)$ on plane so

$$6(0) - 5(-40) + 2(0) = d$$

$$200 = d$$

Plane is

$$6x - 5y + 2z = 200$$

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Normals are $\perp$ so planes are $\perp$

2)

Normal to $2x + 3y + 4z = 10$

is $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

Normal to $x - 2y + z = 5$

is $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = 2 - 6 + 4 = 0$

5)

$$\underline{r} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\3\end{pmatrix}$$

By inspection, when $\lambda = 2$

$$r = \begin{pmatrix}1-2\\2+4\\-1+6\end{pmatrix} = \begin{pmatrix}-1\\6\\5\end{pmatrix}$$

$\therefore (-1, 6, 5)$ is on line

$$\underline{r} = \begin{pmatrix}0\\6\\3\end{pmatrix} + \mu\begin{pmatrix}1\\0\\-2\end{pmatrix}$$

By inspection, when $\lambda = -1$

$$\underline{r} = \begin{pmatrix}0-1\\6+0\\3-2(-1)\end{pmatrix} = \begin{pmatrix}-1\\6\\5\end{pmatrix}$$

$\therefore (-1, 6, 5)$ is on line

Angle between lines is angle between direction vectors

$$\cos\theta = \frac{\begin{pmatrix}-1\\2\\3\end{pmatrix}.\begin{pmatrix}1\\0\\-2\end{pmatrix}}{\left|\begin{pmatrix}-1\\2\\3\end{pmatrix}\right|\left|\begin{pmatrix}1\\0\\-2\end{pmatrix}\right|}$$

$$\cos\theta = \frac{-1+0-6}{\sqrt{(-1)^2+2^2+3^2}\sqrt{1^2+0^2+(-2)^2}}$$

$$\cos\theta = \frac{-7}{\sqrt{14}\ \sqrt{5}} = \frac{-7}{\sqrt{70}}$$

$$\theta = 146.8^\circ$$

Acute angle between lines

$$= 33.2^\circ$$

7)

i) $C(15, 0, 0)$ $\quad D(9, 6, 24)$ $\quad \overrightarrow{CD} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}$

$C(15, 0, 0)$ $\quad B(15, 20, 0)$ $\quad \overrightarrow{CB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$

7ii) $|CD| = \sqrt{(15-9)^2 + (0-6)^2 + (0-24)^2}$

$|CD| = 25.46$ cm

7iii) $\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix} = -24 + 0 + 24 = 0$

$\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} = 0 + 0 + 0 = 0$

$\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$ is $\perp$ to two non-parallel vectors in plane BCDE

It is $\therefore$ normal to plane.

Eqn of form $4x + 0y + z = d$

C on plane so $4(15) + 0(0) + 0 = d$

$60 = d$

Plane is $4x + z = 60$

7iv) For OG

$\underline{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 24 \end{pmatrix}$ or $\underline{r} = \lambda \begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix}$

7iv) cont) For AF $\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$

or $\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix}$

For OG with $\lambda = 5$

$$\underline{r} = 5\begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 40 \end{pmatrix}$$

For AF with $\mu = 5$

$$\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + 5\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 40 \end{pmatrix}$$

$\therefore$ OG and AF meet at $(5, 10, 40)$

7v) Volume of Pyramid POABC

$= \frac{1}{3} \times$ Area of Base $\times$ height

$= \frac{1}{3} \times (15 \times 20) \times 40$

$= 4000 \text{ cm}^3$

Volume of Pyramid PDEFG

$= \frac{1}{3} \times (6 \times 8) \times (40 - 24)$

$= 256 \text{ cm}^3$

Volume of ornament

$= 4000 - 256 = 3744 \text{ cm}^3$

8)

Subst A, B, C in eqn of plane

i) to verify it is plane ABC

$$3x + 2y + 20z + 300 = 0$$

$A(0, 0, -15)$

$$0 + 0 + 20(-15) + 300 = 0 \checkmark$$

$B(100, 0, -30)$

$$300 + 0 + 20(-30) + 300 = 0$$
$$300 - 600 + 300 = 0 \checkmark$$

$C(0, 100, -25)$

$$0 + 2(100) + 20(-25) + 300 = 0$$

$$200 - 500 + 300 = 0 \checkmark$$

$\therefore$ Plane is the plane ABC

ii)

$D(0, 0, -40)$
$E(100, 0, -50)$
$F(0, 100, -35)$

$$\overrightarrow{DE} = \begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} \qquad \overrightarrow{DF} = \begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 100 \\ 0 \\ -10 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = 200 - 200 = 0$$

$$\begin{pmatrix} 0 \\ 100 \\ 5 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix} = -100 + 100 = 0$$

$\therefore \begin{pmatrix} 2 \\ -1 \\ 20 \end{pmatrix}$ is $\perp$ to both $\overrightarrow{DE}$ and $\overrightarrow{DF}$

It is $\therefore$ normal to plane DEF

Plane is of form

$$2x - y + 20z = d$$

$D(0, 0, -40)$ on plane so

$$0 - 0 + 20(-40) = d$$

$$-800 = d$$

Plane DEF is

$$2x - y + 20z = -800$$

8iii) Angle between planes
= angle between their normals

$$\cos\theta = \frac{\begin{pmatrix}3\\2\\20\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\20\end{pmatrix}}{\left|\begin{pmatrix}3\\2\\20\end{pmatrix}\right|\left|\begin{pmatrix}2\\-1\\20\end{pmatrix}\right|}$$

$$\cos\theta = \frac{6-2+400}{\sqrt{9+4+400}\sqrt{4+1+400}}$$

$$\cos\theta = \frac{404}{\sqrt{413}\sqrt{405}}$$

$$\theta = 8.95^\circ$$

8iv)

$$\underline{r} = \begin{pmatrix}15\\34\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\2\\20\end{pmatrix}$$

is eqn of line RS

At S

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}15+3\lambda\\34+2\lambda\\0+20\lambda\end{pmatrix}$$

S on plane ABC so

$$3(15+3\lambda)+2(34+2\lambda)$$
$$+20(20\lambda)+300=0$$

$$45+9\lambda+68+4\lambda+400\lambda+300=0$$

$$413\lambda = -413$$

$$\lambda = -1$$

$$\underline{r}_S = \begin{pmatrix}15-3\\34-2\\-20\end{pmatrix} = \begin{pmatrix}12\\32\\-20\end{pmatrix}$$

S is point $(12, 32, -20)$

3) $\underline{a} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

Require $\lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$

$$2\lambda + 4\mu = 0$$

$$2\lambda = -4\mu$$

$$\lambda = -2\mu$$

Subst in 2nd row

$$-2\mu - 2\mu = 4$$

$$\Rightarrow \quad \mu = -1$$

$$\therefore \quad \lambda = -2(-1) = 2$$

Check in third row

$$2(-1) - 1(1) = -3 \quad \checkmark$$

Answer $\lambda = 2, \ \mu = -1$

5) i) Plane $2x - y + z = 2$

Normal $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Plane $x - z = 1$

Normal $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Angle between planes = angle between normals

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right|}$$

$$\cos\theta = \frac{2 + 0 - 1}{\sqrt{4+1+1}\sqrt{1+0+1}}$$

$$\cos\theta = \frac{1}{\sqrt{12}}$$

$$\theta = 73.2^\circ$$

5ii) $$\underline{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 + 2\lambda \\ -\lambda \\ 1 + \lambda \end{pmatrix}$$

Subst in plane

$$2(2 + 2\lambda) - (-\lambda) + (1 + \lambda) = 2$$

$$4 + 4\lambda + \lambda + 1 + \lambda = 2$$

$$6\lambda = -3$$

$$\lambda = -\tfrac{1}{2}$$

Subst back in line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 + 2(-\frac{1}{2}) \\ -(-\frac{1}{2}) \\ 1 + (-\frac{1}{2}) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Point of intersection $(1, \tfrac{1}{2}, \tfrac{1}{2})$

$$= \frac{1}{4}\int_0^{\frac{\pi}{3}} \sec^2\left(\theta - \tfrac{\pi}{3}\right) d\theta$$

$$(\cot\theta + 1)(\cot\theta - 2) = 0$$

Either $\cot\theta = -1$ or $\cot\theta = 2$

When $\cot\theta = -1$, $\tan\theta = -1$
$\Rightarrow \theta = 135^\circ$

When $\cot\theta = 2$, $\tan\theta = \frac{1}{2}$
$\Rightarrow \theta = 26.6^\circ$

Solution $\theta = 26.6^\circ$, $\theta = 135^\circ$

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7) i) $A(1, 2, 2)$
$B(0, 0, 2)$

$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$$

line AB given by

$$\underline{r} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$$

7ii) normal is $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Find angle between $\overrightarrow{AB}$ and normal

$$\cos\theta = \frac{\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right|}$$

$$\cos\theta = \frac{-1 + 0 + 0}{\sqrt{5}\sqrt{2}} = \frac{-1}{\sqrt{10}}$$

$\theta = 108.4^\circ$

Acute angle is therefore 71.6°

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7iii) Line BC $\underline{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$

$$\cos\phi = \frac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}\right|}$$

$$\cos\phi = \frac{-2 + 0 - 1}{\sqrt{2}\ \sqrt{9}} = \frac{-3}{3\sqrt{2}}$$

$\phi = 135°$

Acute angle $\therefore$ $180 - 135 = 45°$

7iv)

$\sin\theta = k \sin\phi$

$\sin 71.6 = k \sin 45$

$$k = \frac{\sin 71.6°}{\sin 45°} = 1.34$$

7v) $\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 - 2\mu \\ 0 - 2\mu \\ 2 - \mu \end{pmatrix}$

Subst in plane $x + z = -1$

$-2\mu + 2 - \mu = -1$

$-3\mu = -3$

$\mu = 1$

Subst back in line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$$

Point of intersection $(-2, -2, 1)$

Distance through glass

$= \sqrt{(-2-0)^2 + (-2-0)^2 + (1-2)^2}$

$= \sqrt{4 + 4 + 1} = 3\text{ cm}$