1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence find the range of the function $f(\theta)$, where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

Write down the greatest possible value of $\dfrac{1}{7 + 3\cos\theta + 4\sin\theta}$. [6]

3 Solve the equation

$$\sec^2\theta = 4, \quad 0 \leqslant \theta \leqslant \pi,$$

giving your answers in terms of π. [4]

5 Solve the equation $2\cos 2x = 1 + \cos x$, for $0° \leqslant x < 360°$. [7]

4 Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

1 Fig. 1 shows part of the graph of $y = \sin x - \sqrt{3}\cos x$.

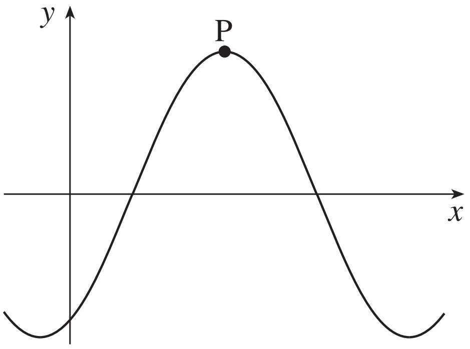

Fig. 1

Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 \leqslant \alpha \leqslant \frac{1}{2}\pi$.

Hence write down the exact coordinates of the turning point P. [6]

3 Given that $\sin(\theta + \alpha) = 2\sin\theta$, show that $\tan\theta = \dfrac{\sin\alpha}{2 - \cos\alpha}$.

Hence solve the equation $\sin(\theta + 40°) = 2\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [7]

3 **(i)** Use the formula for $\sin(\theta + \phi)$, with $\theta = 45°$ and $\phi = 60°$, to show that $\sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}}$. [4]

(ii) In triangle ABC, angle BAC = 45°, angle ACB = 30° and AB = 1 unit (see Fig. 3).

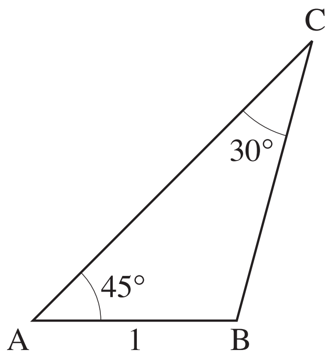

Fig. 3

Using the sine rule, together with the result in part (**i**), show that $AC = \frac{\sqrt{3}+1}{\sqrt{2}}$. [3]

4 Show that $\frac{1+\tan^2\theta}{1-\tan^2\theta} = \sec 2\theta$.

Hence, or otherwise, solve the equation $\frac{1+\tan^2\theta}{1-\tan^2\theta} = 2$, for $0° \leqslant \theta \leqslant 180°$. [7]

1 Express $\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants to be determined, and $0° < \alpha < 90°$.

Hence solve the equation $\sin\theta - 3\cos\theta = 1$ for $0° \leqslant \theta \leqslant 360°$. [7]

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$. [7]

4 The angle θ satisfies the equation $\sin(\theta + 45^\circ) = \cos\theta$.

(i) Using the exact values of $\sin 45^\circ$ and $\cos 45^\circ$, show that $\tan\theta = \sqrt{2} - 1$. [5]

(ii) Find the values of θ for $0^\circ < \theta < 360^\circ$. [2]

6 Solve the equation $\operatorname{cosec}\theta = 3$, for $0^\circ < \theta < 360^\circ$. [3]

3 Solve the equation $\cos 2\theta = \sin \theta$ for $0 \leqslant \theta \leqslant 2\pi$, giving your answers in terms of π. [7]

4 Given that $x = 2 \sec \theta$ and $y = 3 \tan \theta$, show that $\frac{x^2}{4} - \frac{y^2}{9} = 1$. [3]

7 Express $\sqrt{3} \sin x - \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Express α in the form $k\pi$.

Find the exact coordinates of the maximum point of the curve $y = \sqrt{3} \sin x - \cos x$ for which $0 < x < 2\pi$. [6]

4 Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha - \beta)}{\sin\alpha\sin\beta}$. **[3]**

6 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,d\theta = \frac{\sqrt{3}}{4}$. **[4]**

1 Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4\cos\theta - \sin\theta = 3$, for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

[7]

6 Given that $\operatorname{cosec}^2\theta - \cot\theta = 3$, show that $\cot^2\theta - \cot\theta - 2 = 0$.

Hence solve the equation $\operatorname{cosec}^2\theta - \cot\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[6]**

8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π.

(i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

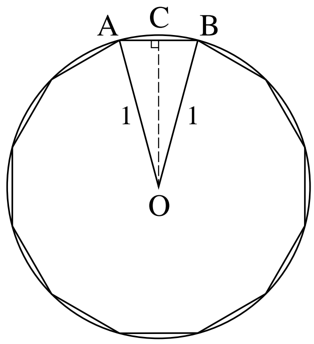

Fig. 8.1

(A) Show that AB = $2\sin 15°$. **[2]**

(B) Use a double angle formula to express $\cos 30°$ in terms of $\sin 15°$. Using the exact value of $\cos 30°$, show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. **[4]**

(C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2-\sqrt{3}}$. **[2]**

(ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

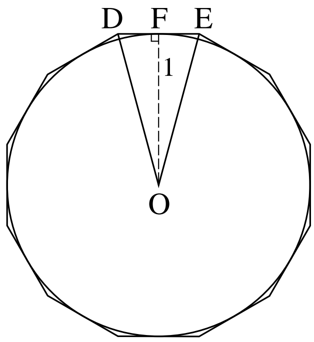

Fig. 8.2

(A) Show that DE = $2\tan 15°$. **[2]**

(B) Let $t = \tan 15°$. Use a double angle formula to express $\tan 30°$ in terms of t.

Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. **[3]**

(C) Solve this equation, and hence show that $\pi < 12\left(2-\sqrt{3}\right)$. **[4]**

(iii) Use the results in parts **(i)**(C) and **(ii)**(C) to establish upper and lower bounds for the value of π, giving your answers in decimal form. **[2]**