6 (i) $y^2 - x^2 = (t + 1/t)^2 - (t - 1/t)^2$ $= t^2 + 2 + 1/t^2 - t^2 + 2 - 1/t^2$ $= 4$	**M1** **E1** **[2]**	Substituting for x and y in terms of t oe
(ii) EITHER $dx/dt = 1 + 1/t^2$, $dy/dt = 1 - 1/t^2$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$ $= \dfrac{1-1/t^2}{1+1/t^2}$ $= \dfrac{t^2-1}{t^2+1} = \dfrac{(t-1)(t+1)}{t^2+1}$ * **OR** $2y\dfrac{dy}{dx} - 2x = 0$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{x}{y} = \dfrac{t-1/t}{t+1/t}$ $= \dfrac{t^2-1}{t^2+1} = \dfrac{(t-1)(t+1)}{t^2+1}$ **OR** $y=\sqrt{(4+x^2)}$, $\Rightarrow \dfrac{dy}{dx} = \dfrac{x}{\sqrt{4+x^2}}$ $= \dfrac{t-1/t}{\sqrt{4+t^2-2+1/t^2}}$ $= \dfrac{t-1/t}{\sqrt{(t+1/t^2)}} = \dfrac{t-1/t}{(t+1/t)}$ $= \dfrac{t^2-1}{t^2+1} = \dfrac{(t-1)(t+1)}{t^2+1}$ $\Rightarrow$ $dy/dx = 0$ when $t = 1$ or -1 $t = 1, \Rightarrow (0, 2)$ $t = -1 \Rightarrow (0, -2)$	**B1** **M1** **E1** **B1** **M1** **E1** **B1** **M1** **E1** **M1** **A1 A1** **[6]**	For both results

2 $dx/dt = 1 - 1/t$ $dy/dt = 1 + 1/t$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1+\frac{1}{t}}{1-\frac{1}{t}}$ When $t = 2$, $dy/dx = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$	B1 M1 A1 M1 A1 [5]	Either dx/dt or dy/dt soi www

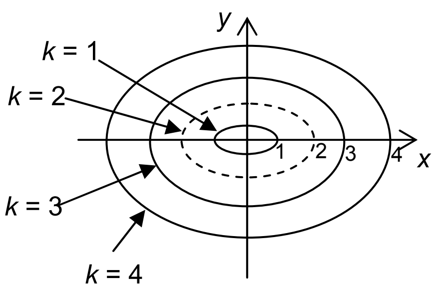
Section B

6 (i) At E, $\theta = 2\pi$ $\Rightarrow$ $x = a(2\pi - \sin 2\pi) = 2a\pi$ So OE = $2a\pi$. Max height is when $\theta = \pi$ $\Rightarrow$ $y = a(1 - \cos \pi) = 2a$	M1 A1 M1 A1 [4]	$\theta=\pi$, 180°, $\cos\theta=-1$
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{a\sin\theta}{a(1-\cos\theta)}$ $= \frac{\sin\theta}{(1-\cos\theta)}$	M1 M1 A1 [3]	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ for theirs $\frac{d}{d\theta}(\sin\theta) = \cos\theta$, $\frac{d}{d\theta}(\cos\theta) = -\sin\theta$ both or equivalent www condone uncancelled a
(iii) $\tan 30° = 1/\sqrt{3}$ $\Rightarrow$ $\frac{\sin\theta}{(1-\cos\theta)} = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$ * When $\theta = 2\pi/3$, $\sin\theta = \sqrt{3}/2$ $(1 - \cos\theta)/\sqrt{3} = (1 + ½)/\sqrt{3} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ BF = $a(1 + ½) = 3a/2$* OF = $a(2\pi/3 - \sqrt{3}/2)$	M1 E1 M1 E1 E1 B1 [6]	Or gradient=$1/\sqrt{3}$ $\sin\theta = \sqrt{3}/2$, $\cos\theta = -½$ or equiv.
(iv) BC = $2a\pi - 2a(2\pi/3 - \sqrt{3}/2)$ $= a(2\pi/3 + \sqrt{3})$ AF = $\sqrt{3} \times 3a/2 = 3\sqrt{3}a/2$ AD = BC + 2AF $= a(2\pi/3 + \sqrt{3} + 3\sqrt{3})$ $= a(2\pi/3 + 4\sqrt{3})$ $= 20$ $\Rightarrow a = 2.22$ m	B1ft M1 A1 M1 A1 [5]	their OE -2their OF

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\frac{dy}{dx} = \frac{10\cos\theta + 10\cos 2\theta}{-10\sin\theta - 10\sin 2\theta}$	M1	$dy/d\theta \div dx/d\theta$
$= -\frac{\cos\theta + \cos 2\theta}{\sin\theta + \sin 2\theta}$ *	E1	
When $\theta = \pi/3$, $\frac{dy}{dx} = -\frac{\cos\pi/3 + \cos 2\pi/3}{\sin\pi/3 + \sin 2\pi/3}$ $= 0$ as $\cos\pi/3 = \frac{1}{2}$, $\cos 2\pi/3 = -\frac{1}{2}$	B1	or solving $\cos\theta + \cos 2\theta = 0$
At A $x = 10\cos\pi/3 + 5\cos 2\pi/3$ $= 2\frac{1}{2}$ $y = 10\sin\pi/3 + 5\sin 2\pi/3 = 15\sqrt{3}/2$	M1 A1 A1 [6]	substituting $\pi/3$ into x or y $2\frac{1}{2}$ $15\sqrt{3}/2$ (condone 13 or better)
(ii) $x^2 + y^2 = (10\cos\theta + 5\cos 2\theta)^2 + (10\sin\theta + 5\sin 2\theta)^2$ $= 100\cos^2\theta + 100\cos\theta\cos 2\theta + 25\cos^2 2\theta$ $+ 100\sin^2\theta + 100\sin\theta\sin 2\theta + 25\sin^2 2\theta$	B1 M1	 expanding
$= 100 + 100\cos(2\theta - \theta) + 25$	DM1	$\cos 2\theta\cos\theta + \sin 2\theta\sin\theta = \cos(2\theta - \theta)$ or substituting for $\sin 2\theta$ and $\cos 2\theta$
$= 125 + 100\cos\theta$ *	E1 [4]	
(iii) Max $\sqrt{125+100} = 15$ min $\sqrt{125-100} = 5$	B1 B1 [2]	
(iv) $2\cos^2\theta + 2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2\pm\sqrt{12}}{4} = \frac{-2\pm 2\sqrt{3}}{4}$	M1	quadratic formula
At B, $\cos\theta = \frac{-1+\sqrt{3}}{2}$	A1	or $\theta = 68.53°$ or 1.20radians, correct root selected
$OB^2 = 125 + 50(-1+\sqrt{3}) = 75 + 50\sqrt{3} = 161.6\ldots$ $\Rightarrow$ $OB = \sqrt{161.6\ldots} = 12.7$ (m)	M1 A1 [4]	or $OB = 10\sin\theta + 5\sin 2\theta$ ft their $\theta/\cos\theta$ oe cao

8(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -\frac{1}{2}\cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	oe
or, by differentiating implicitly $2x + 8y\,dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int \frac{dy}{y} = \int \frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4\ln x\ (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www

5(i) dx/du = 2u , dy/du = 6u^2 $\Rightarrow \frac{dy}{dx} = \frac{dy/du}{dx/du} = \frac{6u^2}{2u}$ $= 3u$ **OR** $y=2(x-1)^{3/2}$, $dy/dx=3(x-1)^{1/2}=3u$	B1 M1 A1 [3]	both 2u and 6u^2 B1(y=f(x)), M1 differentiation, A1
(ii) At (5, 16), u = 2 $\Rightarrow$ dy/dx = 6	M1 A1 [2]	cao

8(i) $\frac{dy}{d\theta}=2\cos 2\theta-2\sin\theta$, $\frac{dx}{d\theta}=2\cos\theta$ $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos 2\theta-2\sin\theta}{2\cos\theta}=\frac{\cos 2\theta-\sin\theta}{\cos\theta}$	B1, B1 M1 A1 [4]	substituting for theirs oe
(ii) When $\theta=\pi/6$, $\frac{dy}{dx}=\frac{\cos\pi/3-\sin\pi/6}{\cos\pi/6}$ $=\frac{1/2-1/2}{\sqrt{3}/2}=0$ Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ BC = $2\times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1,A1 B1ft [5]	for either exact
(iii) (A) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ * (B) $\sin\theta = ½ (x - 2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼ (x - 2)^2$ $= 1 - ¼ x^2 + x - 1$ $= (x - ¼ x^2)$ * (C) Cartesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - ¼ x^2)$ $= x^3 - ¼ x^4$*	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2\sin\theta\cos\theta$ squaring and substituting for x
(iv) $V=\int_0^4 \pi y^2 dx$ $=\pi\int_0^4 (x^3-\frac{1}{4}x^4)dx$ $=\pi\left[\frac{1}{4}x^4-\frac{1}{20}x^5\right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4-\frac{1}{20}x^5\right]$ 12.8π or 40 or better.

5 $\frac{dy}{dt} = -a(1+t^2)^{-2}.2t$ $\frac{dx}{dt} = 3at^2$ $\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at}{3at^2(1+t^2)^2}$ $= \frac{-2}{3t(1+t^2)^2}$ * At $(a, ½\, a)$, $t = 1$ $\Rightarrow$ gradient = $\frac{-2}{3\times 2^2} = -1/6$	M1 A1 B1 M1 E1 M1 A1 [7]	$(1+t^2)^{-2} \times kt$ for method ft finding t