MEI CORE 3 JUNE 2008 SECTION B

8)

i) $y = f(x) = \dfrac{1}{1+\cos x} \qquad 0 \le x \le \frac{\pi}{2}$

At P, $x = \frac{\pi}{3} \Rightarrow y = \dfrac{1}{1+\cos\frac{\pi}{3}} = \dfrac{1}{1+\frac{1}{2}} = \dfrac{2}{3}$

$\therefore P\left(\frac{\pi}{3}, \frac{2}{3}\right)$

ii) $f'(x) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \dfrac{(1+\cos x)0 - 1(-\sin x)}{(1+\cos x)^2}$

$f'(x) = \dfrac{\sin x}{(1+\cos x)^2}$

When $x = \frac{\pi}{3}$ $\quad f'\left(\frac{\pi}{3}\right) = \dfrac{\sin\frac{\pi}{3}}{\left(1+\cos\frac{\pi}{3}\right)^2} = \dfrac{\frac{\sqrt{3}}{2}}{\left(\frac{3}{2}\right)^2}$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{4}{9} = \dfrac{2}{3\sqrt{3}}$

Gradient at P is $\dfrac{2}{3\sqrt{3}}$

iii) $\dfrac{d}{dx}\dfrac{\sin x}{1+\cos x} = \dfrac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \dfrac{\cos x + 1}{(1+\cos x)^2}$

$= \dfrac{1}{1+\cos x}$

Area $= \displaystyle\int_0^{\frac{\pi}{3}} \frac{1}{1+\cos x}\,dx = \left[\frac{\sin x}{1+\cos x}\right]_0^{\frac{\pi}{3}}$

8iii cont)

$$= \left(\frac{\sin\frac{\pi}{3}}{1+\cos\frac{\pi}{3}}\right) - \left(\frac{\sin 0}{1+\cos 0}\right)$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} - 0$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{3} = \frac{1}{\sqrt{3}} \text{ units}^2$$

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iv) Let $y = \frac{1}{1+\cos x}$

swap variables $x = \frac{1}{1+\cos y}$

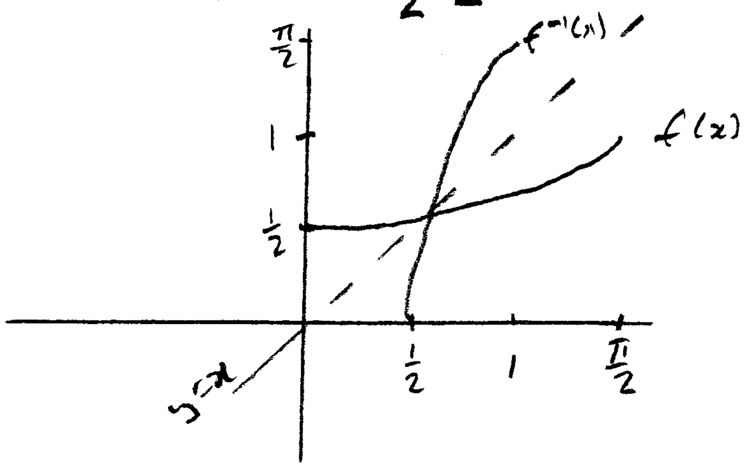
$$(1+\cos y)x = 1$$

$$1+\cos y = \frac{1}{x}$$

$$\cos y = \frac{1}{x} - 1$$

$$y = \cos^{-1}\left(\frac{1}{x}-1\right) = f^{-1}(x)$$

Domain of $f^{-1}(x)$ is $\frac{1}{2} \leq x \leq 1$



MEI CORE 3 JUNE 2008 SECTION B

9) i) $f(x) = \sqrt{4 - x^2}$ $\quad -2 \le x \le 2$

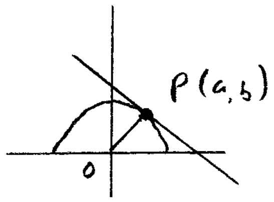
Let $y = \sqrt{4-x^2}$

$y^2 = 4 - x^2 \quad \Rightarrow \quad x^2 + y^2 = 4$

circle centre $(0,0)$ radius 2

However, would need $f(x) = \pm\sqrt{4-x^2}$ for whole circle
Restricting to positive values gives upper semi-circle

ii) A)



gradient $OP = \frac{b-0}{a-0} = \frac{b}{a}$

$\therefore$ gradient of tgt at P is $-\frac{a}{b}$

B) $\frac{d}{dx}\sqrt{4-x^2} = \frac{d}{dx}(4-x^2)^{\frac{1}{2}}$

$= \frac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{4-x^2}}$

$\therefore \quad f'(a) = \frac{-a}{\sqrt{4-a^2}}$

C) On curve $y = \sqrt{4-x^2}$

so at (a,b) $\quad b = \sqrt{4-a^2}$ $\quad \therefore -\frac{a}{b} = \frac{-a}{\sqrt{4-a^2}}$

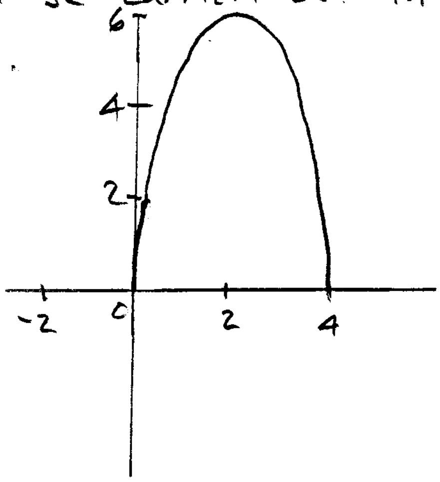
which is the same as $f'(a)$

9) iii) $g(x) = 3f(x-2)$ $\qquad 0 \le x \le 4$

One way stretch parallel to y-axis scale factor 3

Translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

These could be carried out in either order



iv) $y = g(x) = 3f(x-2)$

$y = 3\left(\sqrt{4-(x-2)^2}\right)$

$y^2 = 9\left(4-(x-2)^2\right)$

$y^2 = 9\left(4-(x^2-4x+4)\right)$

$y^2 = 9\left(4-x^2+4x-4\right)$

$y^2 = 9\left(4x-x^2\right)$

$y^2 = 36x - 9x^2$

$9x^2 + y^2 = 36x$