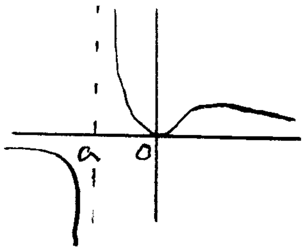
MEI CORE 3 JUNE 2007 SECTION B

7) i)



$$y = \frac{x^2}{1+2x^3}$$

i) Undefined when denominator $=0$

$\Rightarrow 1 + 2x^3 = 0$

$2x^3 = -1$

$x^3 = -\frac{1}{2}$

$x = \sqrt[3]{-\frac{1}{2}}$

$a = -0.794$ to 3 s.f

ii) $\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \dfrac{(1+2x^3)(2x) - x^2(6x^2)}{(1+2x^3)^2}$

$= \dfrac{2x + 4x^4 - 6x^4}{(1+2x^3)^2}$

$= \dfrac{2x - 2x^4}{(1+2x^3)^2}$

At t.p. $\dfrac{dy}{dx} = 0 \Rightarrow$ $2x - 2x^4 = 0$

$2x(1 - x^3) = 0$

$\Rightarrow x = 0$ or $1 - x^3 = 0$

$1 = x^3$

$1 = x$

when $x = 0$, $y = 0$

when $x = 1$

$y = \dfrac{1}{1+2} = \dfrac{1}{3}$

turning points $(0,0)$ and $(1, \frac{1}{3})$

MEI CORE 3   JUNE 2007   SECTION B

7 iii) Area $= \int_0^1 \frac{x^2}{1+2x^3}\,dx$

$$= \frac{1}{6}\int_0^1 \frac{6x^2}{1+2x^3}\,dx$$

$$= \frac{1}{6}\left[\ln(1+2x^3)\right]_0^1$$

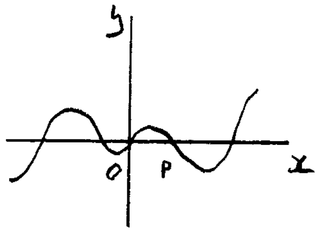
$$= \frac{1}{6}\left[\ln(1+2) - \ln(1+0)\right]$$

$$= \frac{1}{6}\left[\ln 3 - \ln 1\right]$$

$$= \frac{1}{6}\ln 3$$

MEI CORE 3 JUNE 2007 SECTION B

8)



$$y = x\cos 2x$$

i) At P, $x\cos 2x = 0$

$$\Rightarrow x = 0 \quad \text{or} \quad \cos 2x = 0$$

$$\Rightarrow 2x = \frac{\pi}{2}$$

$$x = \frac{\pi}{4}$$

$\therefore P\left(\frac{\pi}{4}, 0\right)$

ii) Let $f(x) = x\cos 2x$

$$f(-x) = (-x)\cos(-2x) = -x\cos 2x = -f(x)$$

$\therefore$ an odd function

It has rotational symmetry of order 2 about $(0,0)$

iii) $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = x(-2\sin 2x) + \cos 2x\,(1)$

$$= -2x\sin 2x + \cos 2x$$

iv) At t.p $\frac{dy}{dx} = 0 \Rightarrow -2x\sin 2x + \cos 2x = 0$

$$\cos 2x = 2x\sin 2x$$

$$1 = 2x\frac{\sin 2x}{\cos 2x}$$

$$\frac{1}{2} = x\tan 2x$$

MEI CORE 3 JUNE 2007 SECTION B

8)

v) when $x = 0$, $\frac{dy}{dx} = -2 \times 0 \sin 0 + \cos 0$

$= 0 + 1$

gradient $= 1$

$$\frac{dy}{dx} = -2x \sin 2x + \cos 2x$$

$$\frac{d^2y}{dx^2} = -2x(2\cos 2x) + \sin 2x(-2) - 2\sin 2x$$

$$\frac{d^2y}{dx^2} = -4x\cos 2x - 4\sin 2x$$

when $x = 0$ $\frac{d^2y}{dx^2} = -4 \times 0 \times \cos 0 - 4\sin 0$

$= 0 - 0$

$= 0$

vi) $\int_0^{\frac{\pi}{4}} x \cos 2x \, dx$

Let $u = x \Rightarrow \frac{du}{dx} = 1$ Let $\frac{dv}{dx} = \cos 2x \Rightarrow v = \frac{1}{2}\sin 2x$

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$\int_0^{\frac{\pi}{4}} x\cos 2x \, dx = \left[\frac{1}{2}x \sin 2x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{2}\sin 2x \, dx$$

$$= \left[\frac{1}{2}x\sin 2x\right]_0^{\frac{\pi}{4}} - \left[-\frac{1}{4}\cos 2x\right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{2} \times \frac{\pi}{4}\sin\frac{\pi}{2} - 0\right) - \left(-\frac{1}{4}\cos\frac{\pi}{2} - -\frac{1}{4}\cos 0\right)$$

$$= \frac{\pi}{8} - \left(0 + \frac{1}{4}\right)$$

$= \frac{\pi}{8} - \frac{1}{4}$ which is area enclosed by curve and x-axis between O and P