MEI CORE 3 JUNE 2006 SECTION B

7)

i) Asymptote $x = 1$

ii) $y = \dfrac{x^2+3}{x-1}$

$$\frac{dy}{dx} = \frac{(x-1)(2x) - (x^2+3)(1)}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 - 2x - x^2 - 3}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 2x - 3}{(x-1)^2}$$

At P, $\dfrac{dy}{dx} = 0 \Rightarrow x^2 - 2x - 3 = 0$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

From graph P has x-coord $= 3$

When $x = 3$, $y = \dfrac{3^2+3}{3-1} = \dfrac{12}{2} = 6$

$\therefore$ P(3, 6)

iii) $\displaystyle\int_2^3 \left(\frac{x^2+3}{x-1}\right) dx$

Let $u = x - 1$

$\dfrac{du}{dx} = 1$

$du = dx$

When $x = 3$, $u = 2$

$x = 2$, $u = 1$

Also $x = u + 1$

$$= \int_1^2 \left(\frac{(u+1)^2+3}{u}\right) du$$

$$= \int_1^2 \left(\frac{u^2+2u+1+3}{u}\right) du$$

$$= \int_1^2 \left(u + 2 + \frac{4}{u}\right) du$$

7iii cont)

$$= \left[\frac{u^2}{2} + 2u + 4\ln u \right]_1^2$$

$$= \left(\frac{2^2}{2} + 2(2) + 4\ln 2 \right) - \left(\frac{1^2}{2} + 2(1) + 4\ln 1 \right)$$

$$= 2 + 4 + 4\ln 2 - \left(\frac{1}{2} + 2 + 0 \right)$$

$$= \frac{7}{2} + 4\ln 2$$

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iv)

$$e^y = \frac{x^2+3}{x-1}$$

When $x = 2$

$$e^y = \frac{2^2+3}{2-1} = 7$$

$$e^y \frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2}$$

$$\frac{dy}{dx} = \frac{x^2-2x-3}{(x-1)^2 e^y}$$

when $x = 2$

$$\frac{dy}{dx} = \frac{2^2-2(2)-3}{(2-1)^2(7)}$$

$$\frac{dy}{dx} = \frac{4-4-3}{7}$$

$$\text{gradient} = -\frac{3}{7}$$

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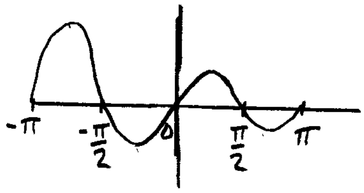
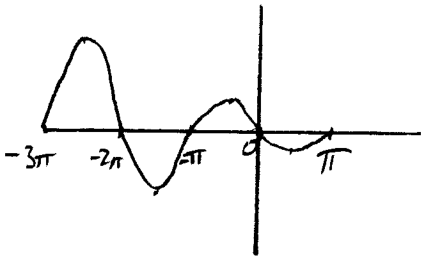
8) i) A)



B)



ii) $f(x) = e^{-\frac{1}{5}x}\sin x$

$f'(x) = e^{-\frac{1}{5}x}\cos x + \sin x\left(-\frac{1}{5}e^{-\frac{1}{5}x}\right)$

$f'(x) = e^{-\frac{1}{5}x}\left(\cos x - \frac{1}{5}\sin x\right)$

At P, $f'(x) = 0$

$\Rightarrow \cos x - \frac{1}{5}\sin x = 0$

$\Rightarrow 5\cos x - \sin x = 0$

$\Rightarrow 5\cos x = \sin x$

$\Rightarrow 5 = \frac{\sin x}{\cos x}$

$\Rightarrow \tan x = 5$

$x = \tan^{-1} 5 = 1.3734$

$y = e^{-\frac{1}{5}(1.3734)}\sin(1.3734) = 0.74506$

$P(1.373, 0.745)$

iii) $f(x+\pi) = e^{-\frac{1}{5}(x+\pi)}\sin(x+\pi)$

$= e^{-\frac{1}{5}x} \times e^{-\frac{\pi}{5}}(-\sin x)$

$= -e^{-\frac{\pi}{5}}\left(e^{-\frac{1}{5}x}\sin x\right) = -e^{-\frac{\pi}{5}}f(x)$

8 iii (cont)

Let $u = x - \pi$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$x = u + \pi$$

when $x = 2\pi, \ u = \pi$
$x = \pi, \ u = 0$

$$\int_{\pi}^{2\pi} f(x)\,dx = \int_0^{\pi} f(u+\pi)\,du$$

$$= \int_0^{\pi} \left(-e^{-\frac{1}{5}\pi} f(u)\right) du$$

$$= -e^{-\frac{1}{5}\pi} \int_0^{\pi} f(u)\,du$$

The areas between successive loops and the x-axis are reduced by a multiple of $e^{-\frac{1}{5}\pi}$

The minus sign indicates the oscillation of the areas being above and below the x-axis