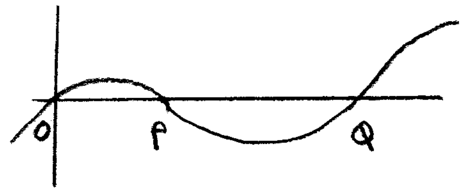
MEI CORE 3 JANUARY 2010 SECTION B

8) i) $y = x\cos 3x$



At P, Q $\quad x\cos 3x = 0$

$\Rightarrow x = 0$ or $\cos 3x = 0$

$\Rightarrow 3x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow x = \frac{\pi}{6}, \frac{\pi}{2}$

$P\left(\frac{\pi}{6}, 0\right) \qquad Q\left(\frac{\pi}{2}, 0\right)$

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ii) $\frac{dy}{dx} = x(-3\sin 3x) + \cos 3x(1) \qquad = -3x\sin 3x + \cos 3x$

At P $\quad \frac{dy}{dx} = -3\frac{\pi}{6}\sin\frac{3\pi}{6} + \cos\frac{3\pi}{6}$

$= -\frac{\pi}{2} + 0 \qquad\qquad = -\frac{\pi}{2}$

At t.p.s $\quad \frac{dy}{dx} = 0 \qquad \Rightarrow -3x\sin 3x + \cos 3x = 0$

$\cos 3x = 3x\sin 3x$

$1 = 3x\frac{\sin 3x}{\cos 3x}$

$\frac{1}{3} = x\tan 3x$

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iii) $$\int_0^{\frac{\pi}{6}} x\cos 3x\,dx$$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \cos 3x$ $\Rightarrow v = \frac{1}{3}\sin 3x$

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$\int_0^{\frac{\pi}{6}} x\cos 3x\,dx = \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \frac{1}{3}\sin 3x\,dx$$

$$= \left[\frac{1}{3}x\sin 3x\right]_0^{\frac{\pi}{6}} - \left[-\frac{1}{9}\cos 3x\right]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{1}{3}\cdot\frac{\pi}{6}\sin\frac{\pi}{2} - 0\right] - \left[-\frac{1}{9}\cos\frac{3\pi}{6} - -\frac{1}{9}\cos 0\right]$$

$$= \frac{\pi}{18} - \left[0 + \frac{1}{9}\right]$$

$$= \frac{\pi}{18} - \frac{1}{9}$$

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9) i) $y = f(x) = \dfrac{2x^2-1}{x^2+1}$ $\quad 0 \le x \le 2$

$$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(x^2+1)(4x) - (2x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{4x^3 + 4x - 4x^3 + 2x}{(x^2+1)^2}$$

$$= \frac{6x}{(x^2+1)^2} \quad > 0 \text{ for } x > 0$$

Gradient positive for positive x so an increasing function

ii) When $x = 0$, $f(0) = \dfrac{-1}{+1} = -1$

When $x = 2$ $\quad f(2) = \dfrac{2(2)^2-1}{2^2+1} = \dfrac{7}{5}$

Range $-1 \le f(x) \le \dfrac{7}{5}$

iii) Max $f'(x)$ when $f''(x) = 0$

Given $f''(x) = \dfrac{6-18x^2}{(x^2+1)^3}$

$$f''(x) = 0 \Rightarrow 6 - 18x^2 = 0$$

$$6 = 18x^2$$

$$\frac{1}{3} = x^2$$

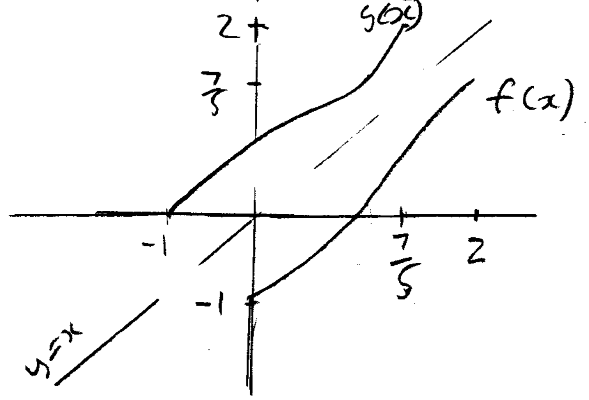
$$\frac{1}{\sqrt{3}} = x \quad (\text{since } x > 0)$$

9iii) cont) When $x = \frac{1}{\sqrt{3}}$ $\quad f'(x) = \dfrac{6\left(\frac{1}{\sqrt{3}}\right)}{\left(\left(\frac{1}{\sqrt{3}}\right)^2+1\right)^2} = \dfrac{6/\sqrt{3}}{\left(\frac{4}{3}\right)^2}$

$$= \frac{6}{\sqrt{3}} \times \frac{9}{16} = \frac{9\sqrt{3}}{8}$$

iv) Domain of $g(x)$ $\quad -1 \leq x \leq \frac{7}{5}$

Range of $g(x)$ $\quad 0 \leq g(x) \leq 2$



v) Let $y = f(x) = \dfrac{2x^2-1}{x^2+1}$

Swap Variables $\quad x = \dfrac{2y^2-1}{y^2+1}$

$$x(y^2+1) = 2y^2-1$$

$$xy^2+x = 2y^2-1$$

$$x+1 = 2y^2-xy^2$$

$$x+1 = y^2(2-x)$$

$$\frac{x+1}{2-x} = y^2$$

$$\sqrt{\frac{x+1}{2-x}} = y \qquad \therefore g(x) = \sqrt{\frac{x+1}{2-x}}$$