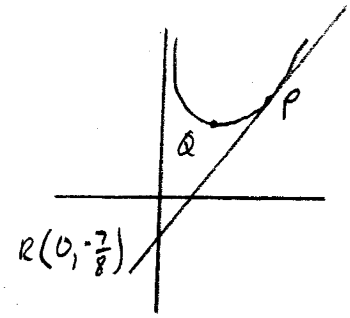
MEI CORE 3   JANUARY 2009   SECTION B

8) i) $y = x^2 - \frac{1}{8}\ln x$



when $x = 1$

$y = 1^2 - \frac{1}{8}\ln 1 = 1$

$\therefore P(1, 1)$

Gradient of PR $= \dfrac{1 - -\frac{7}{8}}{1 - 0} = \dfrac{15}{8}$

ii) $\dfrac{dy}{dx} = 2x - \dfrac{1}{8x}$ At $P(1,1)$

$\dfrac{dy}{dx} = 2(1) - \dfrac{1}{8(1)} = \dfrac{15}{8}$

$\therefore$ PR is a tangent to curve as it has same gradient at P

iii) At t.p Q $\dfrac{dy}{dx} = 0 \Rightarrow 2x - \dfrac{1}{8x} = 0$

$16x^2 - 1 = 0$

$16x^2 = 1$

$x^2 = \frac{1}{16}$

$x = \frac{1}{4}$

When $x = \frac{1}{4}$, $y = \left(\frac{1}{4}\right)^2 - \frac{1}{8}\ln\left(\frac{1}{4}\right) = \dfrac{1}{16} + \dfrac{1}{8}\ln 4$

$= \dfrac{1}{16} + \dfrac{1}{4}\ln 2$

$Q\left(\frac{1}{4}, \frac{1}{16} + \frac{1}{4}\ln 2\right)$

8)

iv) $\frac{d}{dx}(x\ln x - x) = x\frac{1}{x} + \ln x \times 1 - 1$

$= 1 + \ln x - 1 \quad = \ln x$

$$\int_1^2 \left(x^2 - \tfrac{1}{8}\ln x\right)dx$$

$$= \left[\frac{x^3}{3} - \frac{1}{8}(x\ln x - x)\right]_1^2$$

$$= \left[\frac{x^3}{3} - \frac{1}{8}x\ln x + \frac{x}{8}\right]_1^2$$

$$= \left(\frac{8}{3} - \frac{1}{8}\times 2\ln 2 + \frac{2}{8}\right) - \left(\frac{1}{3} - \frac{1}{8}\times 1 \times \ln 1 + \frac{1}{8}\right)$$

$$= \frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4} - \frac{1}{3} + 0 - \frac{1}{8}$$

$$= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$$

$$= \frac{56}{24} + \frac{3}{24} - \frac{1}{4}\ln 2$$

$$= \frac{59}{24} - \frac{1}{4}\ln 2$$

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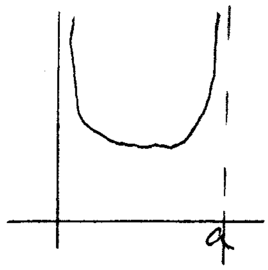
9) i) $y = f(x) = \dfrac{1}{\sqrt{2x - x^2}}$



$f(x) = \dfrac{1}{\sqrt{x(2-x)}}$

$a = 2$

Domain of $f(x)$ $\quad 0 < x < 2$

ii) $y = (2x - x^2)^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = -\dfrac{1}{2}(2x - x^2)^{-3/2}(2 - 2x)$

$= \dfrac{-(1-x)}{(2x-x^2)^{3/2}} = \dfrac{x-1}{(2x-x^2)^{3/2}}$

At t.p, $\dfrac{dy}{dx} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

when $x = 1$, $y = \dfrac{1}{\sqrt{2(1) - 1^2}} = \dfrac{1}{\sqrt{2-1}} = 1$

turning point at $(1, 1)$ $\quad$ Range $y \geqslant 1$

iii) $g(x) = \dfrac{1}{\sqrt{1 - x^2}}$

A) $g(-x) = \dfrac{1}{\sqrt{1 - (-x)^2}} = \dfrac{1}{\sqrt{1 - x^2}} = g(x)$ $\therefore$ even

B) $g(x-1) = \dfrac{1}{\sqrt{1 - (x-1)^2}} = \dfrac{1}{\sqrt{1 - (x^2 - 2x + 1)}}$

$= \dfrac{1}{\sqrt{1 - x^2 + 2x - 1}} = \dfrac{1}{\sqrt{2x - x^2}} = f(x)$

9 iii) c) $g(x)$ is even and is therefore symmetrical about y-axis

$f(x)$ is a translation of $g(x)$ by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

so $f(x)$ is also symmetrical with a line of symmetry $x = 1$