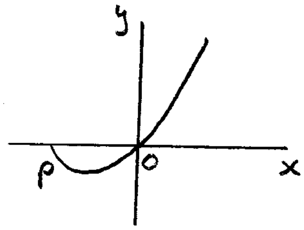
MEI CORE 3 JANUARY 2007 SECTION B

7) $y = f(x) = x\sqrt{1+x}$




i) At P, $y = 0$

$\Rightarrow \quad x\sqrt{1+x} = 0$

$\Rightarrow \quad x = 0$ or $\sqrt{1+x} = 0$

$\Rightarrow \quad 1+x = 0$

$x = -1$

Domain $x \geqslant -1$

$\therefore P(-1, 0)$

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ii) $y = x(1+x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = x \times \frac{1}{2}(1+x)^{-\frac{1}{2}} \times 1 + (1+x)^{\frac{1}{2}} \times 1$$

$$\frac{dy}{dx} = \frac{x}{2\sqrt{1+x}} + \sqrt{1+x}$$

$$\frac{dy}{dx} = \frac{x + 2(1+x)}{2\sqrt{1+x}} = \frac{3x+2}{2\sqrt{1+x}}$$

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iii) At t.p. $\frac{dy}{dx} = 0 \quad \Rightarrow \quad 3x + 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$

When $x = -\frac{2}{3}$ $\quad y = -\frac{2}{3}\sqrt{1-\frac{2}{3}} = -\frac{2}{3\sqrt{3}}$

turning point at $\left(-\frac{2}{3} \quad -\frac{2}{3\sqrt{3}}\right)$

Range $\quad f(x) \geqslant -\frac{2}{3\sqrt{3}}$

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7iv)

$$\int_{-1}^{0} x\sqrt{1+x}\,dx$$

Let $u = 1+x$

$\frac{du}{dx} = 1$

$du = dx$

$x = u-1$

when $x=0,\ u=1$
$x=-1\ u=0$

$$= \int_{0}^{1} (u-1)u^{\frac{1}{2}}\,du$$

$$= \int_{0}^{1} \left(u^{3/2} - u^{\frac{1}{2}}\right)du$$

$$= \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_0^1$$

$$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$$

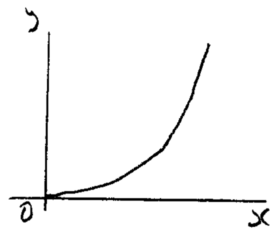
$$= \left(\frac{2}{5} - \frac{2}{3}\right) - (0-0)$$

$$= \frac{6}{15} - \frac{10}{15}$$

$$= -\frac{4}{15}$$

Area $= \frac{4}{15}$ units$^2$ - sign indicates area is below $x$-axis

8) i)




$f(x) = (e^x - 1)^2 \quad x \geqslant 0$

$f'(x) = 2(e^x - 1)e^x$

$= 2e^{2x} - 2e^x$

At $(0, 0)$ $\quad f'(0) = 2e^0 - 2e^0 = 0$

At $(\ln 2, 1)$ $\quad f'(\ln 2) = 2e^{2\ln 2} - 2e^{\ln 2}$

$= 2e^{\ln 2^2} - 2e^{\ln 2}$

$= 2 \times 4 - 2 \times 2$

$= 4$

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ii) $g(x) = \ln(1 + \sqrt{x})$

$gf(x) = g(e^x - 1)^2 = \ln(1 + e^x - 1) = \ln(e^x) = x$

$\therefore$ g and f are inverse functions

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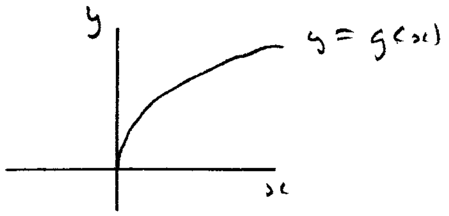


gradient of $g(x)$ at $(1, \ln 2)$

$= \dfrac{1}{\text{grad of } f(x) \text{ at } (\ln 2, 1)} = \dfrac{1}{4}$

<u>OR</u>

$y = (e^x - 1)^2$

swap variables $\quad x = (e^y - 1)^2$

$\sqrt{x} = e^y - 1$

$1 + \sqrt{x} = e^y$

$\ln(1 + \sqrt{x}) = y$

$g(x) = y$ $\qquad \therefore$ f and g are inverse functions

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iii) $\int (e^x - 1)^2 \, dx = \int (e^{2x} - 2e^x + 1) \, dx$

$= \frac{1}{2} e^{2x} - 2e^x + x + c$

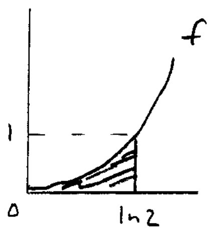
$\int_0^{\ln 2} (e^x - 1)^2 \, dx = \left[\frac{1}{2} e^{2\ln 2} - 2e^{\ln 2} + \ln 2\right] - \left[\frac{1}{2} e^0 - 2e^0 + 0\right]$

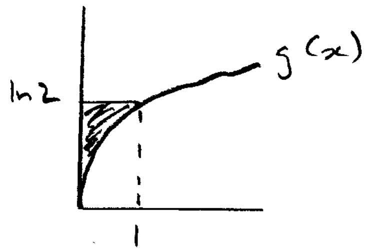
$= \frac{1}{2} e^{\ln 2^2} - 2 \times 2 + \ln 2 - \left[\frac{1}{2} - 2\right]$

$= 2 - 4 + \ln 2 - \frac{1}{2} + 2$

$= \ln 2 - \frac{1}{2}$

iv)




$\ln 2 - \frac{1}{2}$ shaded $\ln 2 - \frac{1}{2}$

Area under $y = g(x)$ between $x = 0$ and $x = 1$

= rectangle − shaded area

$= \ln 2 \times 1 - \left(\ln 2 - \frac{1}{2}\right)$

$= \ln 2 - \ln 2 + \frac{1}{2}$

$= \frac{1}{2}$ unit$^2$