MEI Core 1 Proof Questions Jan 05 - May 09

1 In each case, choose one of the statements

$$P \Rightarrow Q \qquad P \Leftarrow Q \qquad P \Leftrightarrow Q$$

to describe the complete relationship between P and Q.

(i) For n an integer:

P: n is an even number
Q: n is a multiple of 4 [1]

(ii) For triangle ABC:

P: B is a right-angle
Q: $AB^2 + BC^2 = AC^2$ [1]

2 The smallest of three consecutive integers is n.

Write down the other two integers.

Prove that the sum of any three consecutive integers is divisible by 3. [3]

3 n is a positive integer. Show that $n^2 + n$ is always even. [2]

4 In each of the following cases choose one of the statements

$$P \Rightarrow Q \qquad P \Leftrightarrow Q \qquad P \Leftarrow Q$$

to describe the complete relationship between P and Q.

(i) P: $x^2 + x - 2 = 0$
Q: $x = 1$ [1]

(ii) P: $y^3 > 1$
Q: $y > 1$ [1]

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5 The converse of the statement 'P $\Rightarrow$ Q' is 'Q $\Rightarrow$ P'.

Write down the converse of the following statement.

'n is an odd integer $\Rightarrow$ $2n$ is an even integer.'

Show that this converse is false. [2]

6 **(i)** Prove that 12 is a factor of $3n^2 + 6n$ for all even positive integers n. [3]

(ii) Determine whether 12 is a factor of $3n^2 + 6n$ for all positive integers n. [2]

7 Given that n is a positive integer, write down whether the following statements are always true (T), always false (F) or could be either true or false (E).

(i) $2n + 1$ is an odd integer

(ii) $3n + 1$ is an even integer

(iii) n is odd $\Rightarrow$ n^2 is odd

(iv) n^2 is odd $\Rightarrow$ n^3 is even [3]

8 Prove that, when n is an integer, $n^3 - n$ is always even. **[3]**