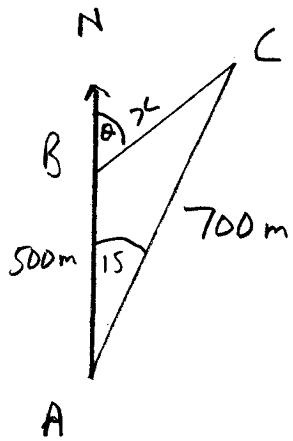
TRIGONOMETRY PROBLEMS 2008-10

6) a)



Let BC be x

Cosine rule

$x^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15°$

$x^2 = 63852$

$x = 252.689$

$x = 253\text{m}$ to 3 s.f.

b) Find θ

First find $\angle ABC$

Sine rule

$$\frac{252.689}{\sin 15°} = \frac{700}{\sin(\angle ABC)}$$

$252.689 \sin(\angle ABC) = 700 \sin 15°$

$$\sin(\angle ABC) = \frac{700 \sin 15°}{252.689}$$

$$\angle ABC = \sin^{-1}\left(\frac{700 \sin 15°}{252.689}\right)$$

$\angle ABC = 45.8°$ or $134.2°$

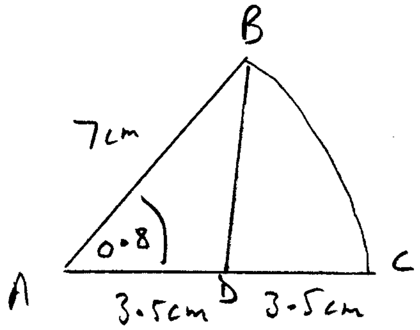
In this case $\angle ABC$ is obtuse $134.2°$

$\theta = 180° - 134.2°$

$\theta = 45.8°$

TRIGONOMETRY PROBLEMS 2008-10

7) a)



Arc length $= r\theta$

$= 7 \times 0.8$

Arc BC $= 5.6$ cm

b) Area of sector ABC $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 7 \times 7 \times 0.8$

$= 19.6\ \text{cm}^2$

c) Find BD using cosine rule

$BD^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5 \cos 0.8$

$BD^2 = 27.111$

$BD = 5.21$ cm

Perimeter $= 5.21 + 3.5 + 5.6$

$= 14.31$ cm

Perimeter of R $= 14.3$ cm to 3 s.f.

d) Area of Δ ABD $= \frac{1}{2} bd \sin A$

$= \frac{1}{2} \times 3.5 \times 7 \sin 0.8 = 8.79\ \text{cm}^2$

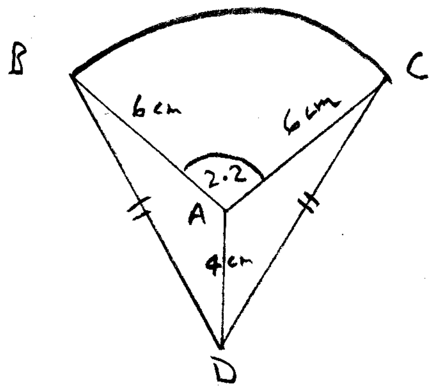
Area of R $= 19.6 - 8.79 = 10.81$

$= 10.8\ \text{cm}^2$ to 3 s.f.

TRIGONOMETRY PROBLEMS

7)

a)



Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 6^2 \times 2.2$

$= 39.6\text{ cm}^2$

b) By symmetry $\angle DAC = \dfrac{2\pi - 2.2}{2} = 2.04$ rad to 3 s.f.

c) Area of $\triangle DAC$ = Area of $\triangle BAD$ $= \frac{1}{2} cd \sin A$

$= \frac{1}{2} \times 4 \times 6 \times \sin 2.04$

$= 10.7\text{ cm}^2$

Area of logo $= 10.7 + 10.7 + 39.6 = 61.0\text{ cm}^2$

TRIGONOMETRY PROBLEMS

9)

a) $$\text{Volume} = \frac{1}{2}r^2\theta h \Rightarrow \frac{1}{2}r^2 \times 1 \times h = 300$$

$$r^2h = 600$$

$$h = \frac{600}{r^2}$$

Surface area =

$$rh + rh + r\theta h + \frac{1}{2}r^2\theta + \frac{1}{2}r^2\theta$$

$$= \frac{600}{r} + \frac{600}{r} + \frac{600 \times 1}{r} + \frac{1}{2}r^2 \times 1 + \frac{1}{2}r^2 1$$

$$= \frac{1800}{r} + r^2$$

b) $$S = 1800r^{-1} + r^2$$

$$\frac{dS}{dr} = -1800r^{-2} + 2r = 2r - \frac{1800}{r^2}$$

S stationary when $\frac{dS}{dr} = 0$ $\Rightarrow 2r - \frac{1800}{r^2} = 0$

$$\Rightarrow 2r^3 - 1800 = 0$$

$$\Rightarrow r^3 = 900$$

$$\Rightarrow r = \sqrt[3]{900} = 9.65\text{cm}$$

c) $$\frac{d^2S}{dr^2} = 2 + 3600r^{-3} = 2 + \frac{3600}{r^3} > 0 \quad \text{when } r = 9.65$$

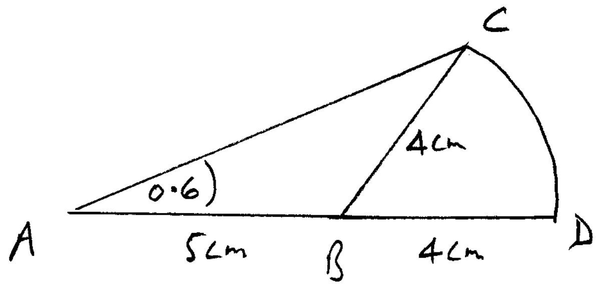
$\therefore$ a minimum

d) $$\text{Min } S = \frac{1800}{9.65} + 9.65^2 = 280\text{ cm}^2 \text{ to 3 s.f.}$$

TRIGONOMETRY PROBLEMS

4)

a)



Sine rule

$$\frac{4}{\sin 0{\cdot}6} = \frac{5}{\sin C}$$

$$\frac{\sin C}{5} = \frac{\sin 0{\cdot}6}{4}$$

$$\sin C = \frac{\sin 0{\cdot}6}{4} \times 5 = 0.7058 \text{ or}$$

$$C = \sin^{-1}(0.7058) = 0.784 \text{ or } \cancel{2.36}$$

since B largest angle

$$\angle ABC = \pi - 0.6 - 0.784 = 1.7576$$

$$= 1.76 \text{ radians to 3 s.f.}$$

b) $\angle CBD = \pi - 1.76 = 1.38$ radians

Area = Area of Δ + Area of sector

$$= \frac{1}{2} \times 5 \times 4 \sin 1.76 + \frac{1}{2} \times 4^2 \times 1.38$$

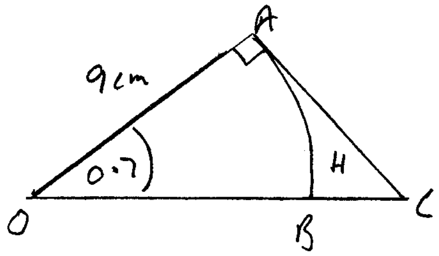
$$= 20.86 \text{ cm}^2$$

$$= 20.9 \text{ cm}^2 \text{ to 3 s.f.}$$

TRIGONOMETRY PROBLEMS

6)

a)



$\text{Arc } AB = r\theta = 9 \times 0.7$

$= 6.3\text{cm}$

b)

$\text{Area of sector} = \frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times 0.7 = 28.35\text{ cm}^2$

c)

$\tan \angle AOC = \frac{AC}{OA}$

$\tan 0.7 = \frac{AC}{9}$

$AC = 9\tan 0.7 = 7.58\text{ cm}$ to 3 s.f.

d)

$\text{Area of } H = \text{Area of } \triangle OAC - \text{Area of sector } OAB$

$= \frac{1}{2} \times 9 \times 7.58 - 28.35 \text{ cm}^2$

$= 5.76\text{ cm}^2$