

# Mark Scheme Summer 2009

GCE

GCE Mathematics (8371/8374; 9371/9374)

Question Number	Scheme	Marks
Q3 (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1} (+ C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1
		(3) [6]
(a)	<p>M1 for an attempt to differentiate <math>x^n \rightarrow x^{n-1}</math></p> <p>1<sup>st</sup> A1 for <math>6x^2</math></p> <p>2<sup>nd</sup> A1 for <math>-6x^{-3}</math> or <math>-\frac{6}{x^3}</math> Condone <math>+ -6x^{-3}</math> here. Inclusion of <math>+c</math> scores A0 here.</p>	
(b)	<p>M1 for some attempt to integrate an <math>x</math> term of the given <math>y</math>. <math>x^n \rightarrow x^{n+1}</math></p> <p>1<sup>st</sup> A1 for <b>both</b> <math>x</math> terms correct but unsimplified- as printed or better. Ignore <math>+c</math> here</p> <p>2<sup>nd</sup> A1 for both <math>x</math> terms correct and simplified and <math>+c</math>. Accept <math>-\frac{3}{x}</math> but <u>NOT</u> <math>+ -3x^{-1}</math></p> <p>Condone the <math>+c</math> appearing on the first (unsimplified) line but missing on the final (simplified) line</p> <p>Apply ISW if a correct answer is seen</p> <p>If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).</p>	

Question Number	Scheme	Marks
Q9 (a)	$\left[ (3 - 4\sqrt{x})^2 = \right] 9 - 12\sqrt{x} - 12\sqrt{x} + (-4)^2 x$ $9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$	M1 A1, A1 (3)
(b)	$f'(x) = -\frac{9}{2}x^{-\frac{3}{2}}, + \frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$	M1 A1 (2)
(a)	<p>M1 for an attempt to expand <math>(3 - 4\sqrt{x})^2</math> with at least 3 terms correct- as printed or better</p> <p><u>Or</u> <math>9 - k\sqrt{x} + 16x</math> (<math>k \neq 0</math>) . See also the MR rule below</p> <p>1<sup>st</sup> A1 for their coefficient of <math>\sqrt{x} = 16</math>. Condone writing <math>(\pm)9x^{(\pm)\frac{1}{2}}</math> instead of <math>9x^{-\frac{1}{2}}</math></p> <p>2<sup>nd</sup> A1 for <math>B = -24</math> or their constant term = -24</p>	
(b)	<p>M1 for an attempt to differentiate an <math>x</math> term <math>x^n \rightarrow x^{n-1}</math></p> <p>1<sup>st</sup> A1 for <math>-\frac{9}{2}x^{-\frac{3}{2}}</math> <u>and</u> their constant <math>B</math> differentiated to zero. NB <math>-\frac{1}{2} \times 9x^{-\frac{3}{2}}</math> is A0</p> <p>2<sup>nd</sup> A1ft follow through their <math>Ax^{\frac{1}{2}}</math> but can be scored without a value for <math>A</math>, i.e. for <math>\frac{A}{2}x^{-\frac{1}{2}}</math></p>	
(c)	<p>M1 for some correct substitution of <math>x = 9</math> in <u>their</u> expression for <math>f'(x)</math> including an attempt at <math>(9)^{\pm\frac{k}{2}}</math> (<math>k</math> odd) somewhere that leads to some appropriate multiples of <math>\frac{1}{3}</math> or 3</p> <p>A1 accept <math>\frac{15}{6}</math> or any exact equivalent of 2.5 e.g. <math>\frac{45}{18}, \frac{135}{54}</math> or even <math>\frac{67.5}{27}</math></p> <p><u>Misread (MR)</u> Only allow MR of the form <math>\frac{(3 - k\sqrt{x})^2}{\sqrt{x}}</math> N.B. Leads to answer in (c) of <math>\frac{k^2 - 1}{6}</math></p> <p>Score as M1A0A0, M1A1A1ft, M1A1ft</p>	

Question Number	Scheme	Marks
Q11 (a) (b) (c)	$x = 2: \quad y = 8 - 8 - 2 + 9 = 7 \quad (*)$ $\frac{dy}{dx} = 3x^2 - 4x - 1$ $x = 2: \quad \frac{dy}{dx} = 12 - 8 - 1 (= 3)$ $y - 7 = 3(x - 2), \quad \underline{y = 3x + 1}$ $m = -\frac{1}{3} \quad \text{(for } -\frac{1}{m} \text{ with their } m)$ $3x^2 - 4x - 1 = -\frac{1}{3}, \quad 9x^2 - 12x - 2 = 0 \quad \text{or} \quad x^2 - \frac{4}{3}x - \frac{2}{9} = 0 \quad \text{(o.e.)}$ $\left( x = \frac{12 + \sqrt{144 + 72}}{18} \right) (\sqrt{216} = \sqrt{36} \cdot \sqrt{6} = 6\sqrt{6}) \quad \text{or} \quad (3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2 + \sqrt{6}) \quad (*)$	B1 (1) M1 A1 A1ft M1, <u>A1</u> (5) B1ft M1, A1 M1 A1cso (5) [11]
(a) (b) (c) ALT	<p>B1 there must be a clear attempt to substitute <math>x = 2</math> leading to 7            e.g. <math>2^3 - 2 \times 2^2 - 2 + 9 = 7</math></p> <p>1<sup>st</sup> M1 for an attempt to differentiate with at least one of the given terms fully correct.            1<sup>st</sup> A1 for a fully correct expression            2<sup>nd</sup> A1ft for sub. <math>x = 2</math> in <u>their</u> <math>\frac{dy}{dx}</math> (<math>\neq y</math>) accept for a correct expression e.g.  <math>3 \times (2)^2 - 4 \times 2 - 1</math></p> <p>2<sup>nd</sup> M1 for use of their “3” (provided it comes from their <math>\frac{dy}{dx}</math> (<math>\neq y</math>) and <math>x=2</math>) to find equation of tangent. Alternative is to use (2, 7) in <math>y = mx + c</math> to <u>find a value</u> for <math>c</math>.            Award when <math>c = \dots</math> is seen.</p> <p><b>No attempted use of <math>\frac{dy}{dx}</math> in (b) scores 0/5</b></p> <p>1<sup>st</sup> M1 for forming an equation from their <math>\frac{dy}{dx}</math> (<math>\neq y</math>) and their <math>-\frac{1}{m}</math> (must be changed from <math>m</math>)            1<sup>st</sup> A1 for a correct 3TQ all terms on LHS (condone missing =0)            2<sup>nd</sup> M1 for proceeding to <math>x = \dots</math> or <math>3x = \dots</math> by formula or completing the square for a 3TQ.            Not factorising. Condone <math>\pm</math>            2<sup>nd</sup> A1 for proceeding to given answer with no incorrect working seen. Can still have <math>\pm</math>.</p> <p><u>Verify (for M1A1M1A1)</u></p> <p>1<sup>st</sup> M1 for attempting to square need <math>\geq 3</math> correct values in <math>\frac{4+6+4\sqrt{6}}{9}</math>, 1<sup>st</sup> A1 for <math>\frac{10+4\sqrt{6}}{9}</math>            2<sup>nd</sup> M1 Dependent on 1<sup>st</sup> M1 in this case for substituting in all terms of their <math>\frac{dy}{dx}</math>            2<sup>nd</sup> A1cso for cso <u>with a full comment</u> e.g. “the <math>x</math> co-ord of <math>Q</math> is ...”</p>	

June 2009  
6664 Core Mathematics C2  
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int \left( 2x + 3x^{\frac{1}{2}} \right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ $\int_1^4 \left( 2x + 3x^{\frac{1}{2}} \right) dx = \left[ x^2 + 2x^{\frac{3}{2}} \right]_1^4 = (16 + 2 \times 8) - (1 + 2)$ $= 29 \quad (29 + C \text{ scores A0})$	<p>M1 A1A1</p> <p>M1</p> <p>A1 (5) [5]</p>
	<p>1<sup>st</sup> M1 for attempt to integrate <math>x \rightarrow kx^2</math> or <math>x^{\frac{1}{2}} \rightarrow kx^{\frac{3}{2}}</math>.</p> <p>1<sup>st</sup> A1 for <math>\frac{2x^2}{2}</math> or a simplified version.</p> <p>2<sup>nd</sup> A1 for <math>\frac{3x^{\frac{3}{2}}}{(\frac{3}{2})}</math> or <math>\frac{3x\sqrt{x}}{(\frac{3}{2})}</math> or a simplified version.</p> <p>Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0.</p> <p>2<sup>nd</sup> M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).</p> <p>Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.</p> <p><u>No working:</u> The answer 29 with no working scores M0A0A0M1A0 (1 mark).</p>	

Question Number	Scheme	Marks
Q9 (a)	<p>(Arc length <math>\Rightarrow r\theta = r \times 1 = r</math>. Can be awarded by implication from later work, e.g. <math>3rh</math> or <math>(2rh + rh)</math> in the <math>S</math> formula. (Requires use of <math>\theta = 1</math>).</p> <p>(Sector area <math>\Rightarrow \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}</math>. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of <math>\theta = 1</math>).</p> <p>Surface area = 2 sectors + 2 rectangles + curved face  <math>(= r^2 + 3rh)</math> (See notes below for what is allowed here)</p> <p>Volume = <math>300 = \frac{1}{2}r^2h</math></p> <p>Sub for <math>h</math>: <math>S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}</math> (*)</p> <p>(b) <math>\frac{dS}{dr} = 2r - \frac{1800}{r^2}</math> or <math>2r - 1800r^{-2}</math> or <math>2r + -1800r^{-2}</math></p> <p><math>\frac{dS}{dr} = 0 \Rightarrow r^3 = \dots</math>, <math>r = \sqrt[3]{900}</math>, or AWR 9.7 (NOT <math>-9.7</math> or <math>\pm 9.7</math>)</p> <p>(c) <math>\frac{d^2S}{dr^2} = \dots</math> and consider sign, <math>\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} &gt; 0</math> so point is a minimum</p> <p>(d) <math>S_{\min} = (9.65\dots)^2 + \frac{1800}{9.65\dots}</math>            (Using their value of <math>r</math>, however found, in the <u>given</u> <math>S</math> formula)  <math>= 279.65\dots</math> (AWRT: 280) (Dependent on full marks in part (b))</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1cso (5)</p> <p>M1A1</p> <p>M1, A1 (4)</p> <p>M1, A1ft (2)</p> <p>M1</p> <p>A1 (2)</p> <p>[13]</p>
(a)	<p>M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an <math>r^2</math> (or <math>r^2\theta</math>) term and an <math>rh</math> (or <math>rh\theta</math>) term.</p> <p>(b) <u>In parts (b), (c) and (d), ignore labelling of parts</u>  <math>1^{\text{st}}</math> M1 for attempt at differentiation (one term is sufficient) <math>r^n \rightarrow kr^{n-1}</math>  <math>2^{\text{nd}}</math> M1 for setting their derivative (a 'changed function') = 0 and solving as far as <math>r^3 = \dots</math> (depending upon their 'changed function', this could be <math>r = \dots</math> or <math>r^2 = \dots</math>, etc., but the algebra <u>must</u> deal with a <u>negative power</u> of <math>r</math> and should be sound apart from possible <u>sign</u> errors, so that <math>r^n = \dots</math> is consistent with their derivative).</p> <p>(c) M1 for attempting second derivative (one term is sufficient) <math>r^n \rightarrow kr^{n-1}</math>, <u>and considering its sign</u>. Substitution of a value of <math>r</math> is not required. (<u>Equating it to zero is M0</u>).            A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. <math>&gt; 0</math>), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.  <u>Alternative:</u>            M1: Find <u>value</u> of <math>\frac{dS}{dr}</math> on each side of their value of <math>r</math> and consider sign.            A1ft: Indicate sign change of negative to positive for <math>\frac{dS}{dr}</math>, and conclude minimum.  <u>Alternative:</u>            M1: Find <u>value</u> of <math>S</math> on each side of their value of <math>r</math> and compare with their 279.65.            A1ft: Indicate that both values are more than 279.65, and conclude minimum.</p>	