CALCULUS JUNE 2008

1)
$$\int (2+5x^2)\,dx = 2x + \frac{5x^3}{3} + c$$

4) a)
$$f(x) = 3x + x^3 \qquad x > 0$$
$$f'(x) = 3 + 3x^2$$

b)
$$f'(x)=15 \Rightarrow 3+3x^2 = 15$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$
But $x > 0$ $\therefore$ $x = 2$

9) a)
$$y = kx^3 - x^2 + x - 5$$
$$\frac{dy}{dx} = 3kx^2 - 2x + 1$$

b) $2y - 7x + 1 = 0$

$2y = 7x - 1$

$y = \frac{7}{2}x - \frac{1}{2}$

$\therefore$ gradient $= \frac{7}{2}$

$$\Rightarrow f'(-\tfrac{1}{2}) = \frac{7}{2}$$
$$\Rightarrow 3k(-\tfrac{1}{2})^2 - 2(-\tfrac{1}{2}) + 1 = \frac{7}{2}$$
$$\frac{3}{4}k + 1 + 1 = \frac{7}{2}$$
$$\frac{3}{4}k = \frac{3}{2}$$
$$3k = 6$$
$$k = 2$$

CALCULUS JUNE 2008

9c) When $x = -\frac{1}{2}$,

$$y = 2\left(-\tfrac{1}{2}\right)^3 - \left(-\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right) - 5$$

$$y = -\frac{1}{4} - \frac{1}{4} - \frac{1}{2} - 5$$

$$y = -6$$

11) a)

$$\frac{dy}{dx} = \frac{(x^2+3)^2}{x^2} = \frac{(x^4 + 6x^2 + 9)}{x^2} \qquad x \neq 0$$

$$= x^2 + 6 + 9x^{-2}$$

b) $(3, 20)$ on C

$$y = \frac{x^3}{3} + 6x + \frac{9x^{-1}}{-1} + c$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} + c$$

Sub $(3, 20)$

$$20 = \frac{3^3}{3} + 6(3) - \frac{9}{3} + c$$

$$20 = 9 + 18 - 3 + c$$

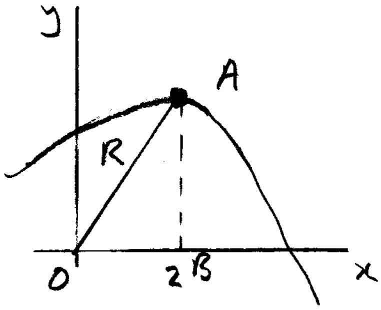
$$-4 = c$$

$$y = \frac{x^3}{3} + 6x - \frac{9}{x} - 4$$

CALCULUS JUNE 2008

8)

a)



$$y = 10 + 8x + x^2 - x^3$$

$$\frac{dy}{dx} = 8 + 2x - 3x^2$$

At t.p $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 8 + 2x - 3x^2 = 0$$

$$\Rightarrow \quad 3x^2 - 2x - 8 = 0$$

$$(3x+4)(x-2) = 0$$

$$\Rightarrow \quad 3x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -\frac{4}{3} \qquad x = 2$$

At A, $x > 0$ $\therefore x = 2$ at max point A

b) At A, $y = 10 + 8(2) + 2^2 - 2^3 = 22$ so $A(2, 22)$

Area of $\triangle OBA = \frac{1}{2} \times 2 \times 22 = 22$ units^2

Area under curve between O and B

$$= \int_0^2 (10 + 8x + x^2 - x^3)\,dx = \left[10x + 4x^2 + \frac{x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \left(10(2) + 4(2)^2 + \frac{2^3}{3} - \frac{2^4}{4}\right) - (0)$$

$$= 20 + 16 + \frac{8}{3} - 4 = 34\tfrac{2}{3}$$

Area of R $= 34\frac{2}{3} - 22 = 12\frac{2}{3}$ units^2