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2) $$\int\left(12x^5-8x^3+3\right)dx = \frac{12x^6}{6}-\frac{8x^4}{4}+3x+c$$

$$= 2x^6-2x^4+3x+c$$

4) $$f'(x) = 3x^2-3x^{\frac{1}{2}}-7$$

$$\Rightarrow \quad f(x) = \frac{3x^3}{3}-\frac{3x^{3/2}}{3/2}-7x+c$$

$$f(x) = x^3-2x^{3/2}-7x+c$$

Through $(4,22)$ $$\Rightarrow \quad 22 = 4^3-2(4)^{3/2}-7(4)+c$$

$$22 = 64-16-28+c$$

$$2 = c$$

$$f(x) = x^3-2x^{3/2}-7x+2$$

6) a) $$\frac{2x^2-x^{3/2}}{\sqrt{x}} = \frac{2x^2-x^{3/2}}{x^{1/2}} = 2x^{3/2}-x^1$$

$$p=\frac{3}{2}, \quad q=1$$

b) $$y = 5x^4-3+2x^{3/2}-x$$

$$\frac{dy}{dx} = 20x^3+\frac{3}{2}\times 2x^{\frac{1}{2}}-1$$

$$\frac{dy}{dx} = 20x^3+3x^{\frac{1}{2}}-1$$

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11) a)

$$y = 9 - 4x - \frac{8}{x} \qquad x > 0$$

$$y = 9 - 4x - 8x^{-1}$$

$$\frac{dy}{dx} = -4 + 8x^{-2}$$

$$\frac{dy}{dx} = \frac{8}{x^2} - 4$$

When $x = 2$, $y = 9 - 4(2) - \frac{8}{2} = -3$ so $P(2, -3)$

When $x = 2$, $\frac{dy}{dx} = \frac{8}{2^2} - 4 = -2$

Tangent using $y - y_1 = m(x - x_1)$

$$y - -3 = -2(x - 2)$$

$$y + 3 = -2x + 4$$

$$y = -2x + 1 \qquad \text{or} \qquad y = 1 - 2x$$

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b) Gradient of normal $= +\frac{1}{2}$

$$y - y_1 = m(x - x_1)$$

$$y - -3 = \frac{1}{2}(x - 2)$$

$$y + 3 = \frac{1}{2}x - 1$$

$$y = \frac{1}{2}x - 4$$

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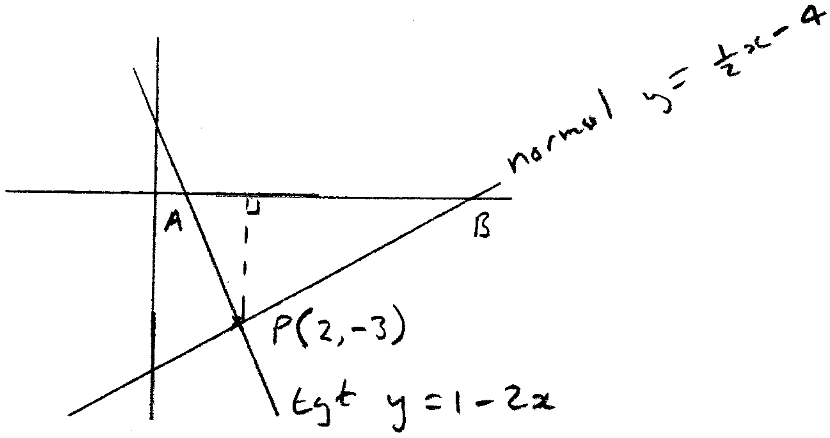
11 c)



At A,

$0 = 1 - 2x$

$2x = 1$

$x = \frac{1}{2}$

At B,

$0 = \frac{1}{2}x - 4$

$4 = \frac{1}{2}x$

$x = 8$

Area of Δ = $\frac{1}{2}$ base $\times$ height

$AB = 8 - \frac{1}{2} = \frac{15}{2}$

height $= 3$

$\frac{1}{2} \times \frac{15}{2} \times 3 = \frac{45}{4}$ or $11\frac{1}{4}$ units^2

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2)

$$y = (1+x)(4-x)$$

$$y = 4 + 4x - x - x^2$$

$$y = 4 + 3x - x^2$$

$$\text{Area } R = \int_{-1}^{4} (4 + 3x - x^2)\,dx$$

$$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^{4}$$

$$= \left(4(4) + \frac{3(4)^2}{2} - \frac{(4)^3}{3}\right) - \left(4(-1) + \frac{3(-1)^2}{2} - \frac{(-1)^3}{3}\right)$$

$$= \left(16 + 24 - \frac{64}{3}\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right)$$

$$= 16 + 24 - \frac{64}{3} + 4 - \frac{3}{2} - \frac{1}{3}$$

$$= \frac{125}{6} \text{ or } 20\tfrac{5}{6} \text{ units}^2$$

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10) a) Volume $= \pi r^2 h$ Surface Area $= 2\pi r^2 + 2\pi r h$

$$800 = 2\pi r^2 + 2\pi r h$$

$$800 - 2\pi r^2 = 2\pi r h$$

$$\frac{800 - 2\pi r^2}{2\pi r} = h$$

$$\text{Volume} = \pi r^2 \left(\frac{800 - 2\pi r^2}{2\pi r} \right) = r\left(\frac{800 - 2\pi r^2}{2} \right)$$

$$= 400r - \pi r^3$$

b) $\frac{dV}{dr} = 400 - 3\pi r^2$ At max $\frac{dv}{dr} = 0$

$$\Rightarrow 400 - 3\pi r^2 = 0$$

$$400 = 3\pi r^2$$

$$\frac{400}{3\pi} = r^2$$

$$r = \sqrt{\frac{400}{3\pi}} = 6.515 \text{ cm}$$

Max V $= 400 \times 6.515 - \pi \times 6.515^3 = 1737 \text{ cm}^3$

c) $\frac{d^2V}{dr^2} = -6\pi r < 0$ for $r = 6.515$ $\therefore$ a maximum