

Mark Scheme (Results) January 2009

GCE

GCE Mathematics (6663/01)

Question Number	Scheme	Marks
2	$(I =) \frac{12}{6}x^6 - \frac{8}{4}x^4 + 3x + c$ $= 2x^6 - 2x^4 + 3x + c$	M1 A1A1A1 [4]
	<p>M1 for an attempt to integrate $x^n \rightarrow x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the $+c$ (seen at some stage), even if no other terms are correct.</p> <p>1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x + c$ (or $3x + k$, etc., any appropriate letter can be used as the constant)</p> <p>Allow $3x^1 + c$, but <u>not</u> $\frac{3x^1}{1} + c$.</p> <p>Note that the A marks can be awarded at separate stages, e.g.</p> $\frac{12}{6}x^6 - 2x^4 + 3x \quad \text{scores 2nd A1}$ $\frac{12}{6}x^6 - 2x^4 + 3x + c \quad \text{scores 3rd A1}$ $2x^6 - 2x^4 + 3x \quad \text{scores 1st A1 (even though the } c \text{ has now been lost).}$ <p>Remember that all the A marks are dependent on the M mark.</p> <p>If applicable, isw (ignore subsequent working) after a correct answer is seen.</p> <p>Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6 - 2x^4 + 3x + c \, dx$.</p>	

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4	$(f(x) =) \frac{3x^3}{3} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 7x(+c)$ $= x^3 - 2x^{\frac{3}{2}} - 7x (+c)$ $f(4) = 22 \Rightarrow 22 = 64 - 16 - 28 + c$ $c = 2$	M1 A1A1 M1 A1cso (5) [5]
	<p>1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The x term is insufficient for this mark and similarly the $+c$ is insufficient.</p> <p>1st A1 for $\frac{3}{3}x^3$ or $-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form)</p> <p>2nd A1 for all three x terms correct and simplified... (the simplification may be seen later). The $+c$ is not required for this mark.</p> <p>Allow $-7x^1$, but <u>not</u> $-\frac{7x^1}{1}$.</p> <p>2nd M1 for an attempt to use $x = 4$ <u>and</u> $y = 22$ in a changed function (even if differentiated) to form an equation in c.</p> <p>3rd A1 for $c = 2$ with no earlier incorrect work (a final expression for $f(x)$ is not required).</p>	

Question Number	Scheme	Marks
6	<p>(a) $2x^{3/2}$ or $p = \frac{3}{2}$ (<u>Not</u> $2x\sqrt{x}$)</p> <p>(b) $-x$ or $-x^1$ or $q = 1$</p> $\left(\frac{dy}{dx} = \right) 20x^3 + 2 \times \frac{3}{2}x^{1/2} - 1$ $= \underline{20x^3 + 3x^{1/2} - 1}$	<p>B1</p> <p>B1 (2)</p> <p>M1</p> <p>A1A1ftA1ft (4)</p> <p>[6]</p>
	<p>(a) 1st B1 for $p = 1.5$ or exact equivalent 2nd B1 for $q = 1$</p> <p>(b) M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ (for any of the 4 terms) 1st A1 for $20x^3$ (the -3 must 'disappear') 2nd A1ft for $3x^{1/2}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a <u>fraction</u>, and the coefficient must be simplified if necessary. 3rd A1ft for -1 (<u>not</u> the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is -1). If it is applied, the coefficient must be simplified if necessary. 'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common factors. Only a single $+$ or $-$ sign is allowed (e.g. $--$ must be replaced by $+$). If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). <u>Multiplying by \sqrt{x}</u>: (assuming this is a restart) e.g. $y = 5x^4\sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$ $\left(\frac{dy}{dx} = \right) \frac{45}{2}x^{7/2} - \frac{3}{2}x^{-1/2} + 4x - \frac{3}{2}x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft. <u>Extra term included</u>: This invalidates the final mark. e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$ $\left(\frac{dy}{dx} = \right) 20x^3 + 4x - \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0. <u>Numerator and denominator differentiated separately</u>: For this, neither of the last two (ft) marks should be awarded. <u>Quotient/product rule</u>: Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)</p>	

Question Number	Scheme	Marks
11 (a)	$\left(\frac{dy}{dx} =\right) -4 + 8x^{-2} \quad (4 \text{ or } 8x^{-2} \text{ for M1... sign can be wrong})$ $x = 2 \Rightarrow m = -4 + 2 = -2$ $y = 9 - 8 - \frac{8}{2} = -3$ <p style="text-align: right;">The first 4 marks <u>could</u> be earned in part (b)</p> <p>Equation of tangent is: $y + 3 = -2(x - 2) \rightarrow y = 1 - 2x \quad (*)$</p> <p>(b) Gradient of normal = $\frac{1}{2}$</p> <p>Equation is: $\frac{y + 3}{x - 2} = \frac{1}{2}$ or better equivalent, e.g. $y = \frac{1}{2}x - 4$</p> <p>(c) (A:) $\frac{1}{2}$, (B:) 8</p> <p>Area of triangle is: $\frac{1}{2}(x_B \pm x_A) \times y_P$ with values for all of x_B, x_A and y_P</p> $\frac{1}{2}\left(8 - \frac{1}{2}\right) \times 3 = \frac{45}{4} \text{ or } 11.25$	M1A1 M1 B1 M1 A1cso (6) B1ft M1A1 (3) B1, B1 M1 A1 (4) [13]
(a)	1 st M1 for 4 or $8x^{-2}$ (ignore the signs). 1 st A1 for both terms correct (including signs). 2 nd M1 for substituting $x = 2$ into their $\frac{dy}{dx}$ (must be different from their y) B1 for $y_P = -3$, but not if clearly found from the given equation of the <u>tangent</u> . 3 rd M1 for attempt to find the equation of tangent at P , follow through their m and y_P . Apply general principles for straight line equations (see end of scheme). <u>NO DIFFERENTIATION ATTEMPTED</u> : Just assuming $m = -2$ at this stage is M0 2 nd A1cso for correct work leading to printed answer (allow equivalents with $2x, y$, and 1 terms... such as $2x + y - 1 = 0$).	
(b)	B1ft for correct use of the perpendicular gradient rule. Follow through their m , but if $m \neq -2$ there must be clear evidence that the m is thought to be the gradient of the tangent. M1 for an attempt to find normal at P using their changed gradient and their y_P . Apply general principles for straight line equations (see end of scheme). A1 for any correct form as specified above (correct answer only).	
(c)	1 st B1 for $\frac{1}{2}$ and 2 nd B1 for 8. M1 for a full method for the area of triangle ABP . Follow through their x_A, x_B and their y_P , but the mark is to be awarded 'generously', condoning sign errors.. The final answer must be positive for A1, with negatives in the working condoned. <u>Determinant</u> : Area = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} 2 & -3 & 1 \\ 0.5 & 0 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \dots$ (Attempt to multiply out required for M1) <u>Alternative</u> : $AP = \sqrt{(2 - 0.5)^2 + (-3)^2}$, $BP = \sqrt{(2 - 8)^2 + (-3)^2}$, Area = $\frac{1}{2} AP \times BP = \dots$ M1 <u>Intersections with y-axis instead of x-axis</u> : Only the M mark is available B0 B0 M1 A0.	

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2	$y = (1 + x)(4 - x) = 4 + 3x - x^2$ <p style="text-align: right;">M: Expand, giving 3 (or 4) terms</p> $\int (4 + 3x - x^2) dx = 4x + \frac{3x^2}{2} - \frac{x^3}{3}$ <p style="text-align: right;">M: Attempt to integrate</p> $= [\dots\dots\dots]_{-1}^4 = \left(16 + 24 - \frac{64}{3}\right) - \left(-4 + \frac{3}{2} + \frac{1}{3}\right) = \frac{125}{6} \quad \left(= 20\frac{5}{6}\right)$	<p>M1</p> <p>M1 A1</p> <p>M1 A1 (5) [5]</p>
Notes	<p>M1 needs expansion, there may be a slip involving a sign or simple arithmetical error e.g. $1 \times 4 = 5$, but there needs to be a ‘constant’ an ‘x term’ and an ‘x^2 term’. The x terms do not need to be collected. (Need not be seen if next line correct)</p> <p>Attempt to integrate means that $x^n \rightarrow x^{n+1}$ for at least one of the terms, then M1 is awarded (even 4 becoming $4x$ is sufficient) – one correct power sufficient.</p> <p>A1 is for correct answer only, not follow through. But allow $2x^2 - \frac{1}{2}x^2$ or any correct equivalent. Allow $+ c$, and even allow an evaluated extra constant term.</p> <p>M1: Substitute limit 4 and limit -1 into a changed function (must be -1) and indicate subtraction (either way round).</p> <p>A1 must be exact, not 20.83 or similar. If recurring indicated can have the mark. Negative area, even if subsequently positive loses the A mark.</p>	
Special cases	<p>(i) Uses calculator method: M1 for expansion (if seen) M1 for limits if answer correct, so 0, 1 or 2 marks out of 5 is possible (Most likely M0 M0 A0 M1 A0)</p> <p>(ii) Uses trapezium rule : not exact, no calculus – 0/5 unless expansion mark M1 gained.</p> <p>(iii) Using original method, but then change all signs after expansion is likely to lead to: M1 M1 A0, M1 A0 i.e. 3/5</p>	

Question Number	Scheme	Marks
<p>10</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	$2\pi rh + 2\pi r^2 = 800$ $h = \frac{400 - \pi r^2}{\pi r}, \quad V = \pi r^2 \left(\frac{400 - \pi r^2}{\pi r} \right) = 400r - \pi r^3 \quad (*)$ $\frac{dV}{dr} = 400 - 3\pi r^2$ $400 - 3\pi r^2 = 0 \quad r^2 = \dots, \quad r = \sqrt{\frac{400}{3\pi}} \quad (= 6.5 \text{ (2 s.f.)})$ $V = 400r - \pi r^3 = 1737 = \frac{800}{3} \sqrt{\frac{400}{3\pi}} \text{ (cm}^3\text{)}$ <p>(accept awrt 1737 or exact answer)</p> $\frac{d^2V}{dr^2} = -6\pi r, \text{ Negative, } \therefore \text{maximum}$ <p>(Parts (b) and (c) should be considered together when marking)</p>	<p>B1</p> <p>M1, M1 A1 (4)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (6)</p> <p>M1 A1 (2)</p> <p>[12]</p>
<p><u>Other methods for part (c):</u></p>	<p><u>Either:</u> M: Find <u>value</u> of $\frac{dV}{dr}$ on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and consider sign.</p> <p>A: Indicate sign change of positive to negative for $\frac{dV}{dr}$, and conclude max.</p> <p><u>Or:</u> M: Find <u>value</u> of V on each side of "$r = \sqrt{\frac{400}{3\pi}}$" and compare with "1737".</p> <p>A: Indicate that both values are less than 1737 or 1737.25, and conclude max.</p>	
<p>Notes</p> <p>(a)</p> <p>(b)</p>	<p>B1: For any correct form of this equation (may be unsimplified, may be implied by 1st M1)</p> <p>M1 : Making h the subject of their three or four term formula</p> <p>M1: Substituting expression for h into $\pi r^2 h$ (independent mark) Must now be expression in r only.</p> <p>A1: cso</p> <p>M1: At least one power of r decreased by 1 A1: cao</p> <p>M1: Setting $\frac{dV}{dr} = 0$ and finding a value for correct power of r for candidate</p> <p>A1 : This mark may be credited if the value of V is correct. Otherwise answers should round to 6.5 (allow ± 6.5) or be exact answer</p> <p>M1: Substitute a positive value of r to give V A1: 1737 or 1737.25..... or exact answer</p>	

<p>Alternative for (a)</p>	<p>(c) M1: needs complete method e.g.attempts differentiation (power reduced) of their first derivative and considers its sign A1(first method) should be $-6\pi r$ (do not need to substitute r and can condone wrong r if found in (b)) Need to conclude maximum or indicate by a tick that it is maximum. Throughout allow confused notation such as dy/dx for dV/dr</p> <p>$A = 2\pi r^2 + 2\pi rh$, $\frac{A}{2} \times r = \pi r^3 + \pi r^2 h$ is M1 Equate to $400r$ B1 Then $V = 400r - \pi r^3$ is M1 A1</p>
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