Mixed Exercise 5

9) $2x - y - 1 = 0$ $\quad l$

a) $2x - 1 = y$ $\quad l$ $\quad$ gradient $= 2$

gradient of $m = -\frac{1}{2}$ $\quad$ thro $A(0,4)$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = -\frac{1}{2}(x - 0)$$
$$\underline{y = -\frac{1}{2}x + 4}$$

b) $\underset{l}{2x - y - 1 = 0}$ $\qquad$ $\underset{m}{y = -\frac{1}{2}x + 4}$

Sub for y in l

$$2x - \left(-\frac{1}{2}x + 4\right) - 1 = 0$$
$$2x + \frac{1}{2}x - 4 - 1 = 0$$
$$4x + x - 8 - 2 = 0$$
$$5x - 10 = 0$$
$$5x = 10$$
$$\underline{x = 2}$$

Sub for x in m $\quad$ $y = -\frac{1}{2}(2) + 4$

$$y = -1 + 4$$
$$\underline{y = 3}$$

Lines l and m intersect at $(2, 3)$

c) n parallel to m $\Rightarrow$ gradient of $n = -\frac{1}{2}$

thro $B(3,0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{1}{2}(x - 3)$$

n $\quad y = -\frac{1}{2}x + \frac{3}{2}$

Sub for y in l

$$2x - \left(-\frac{1}{2}x + \frac{3}{2}\right) - 1 = 0$$

$$2x + \frac{1}{2}x - \frac{3}{2} - 1 = 0$$

$$4x + x - 3 - 2 = 0$$

$$5x - 5 = 0$$

$$5x = 5$$

$$\underline{x = 1}$$

Sub for x in n $\quad y = -\frac{1}{2}(1) + \frac{3}{2}$

$$y = 1$$

l and n intersect at $(1,1)$

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13) $(-1, 5)$ $\quad (4, -2)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y-5}{-2-5} = \frac{x--1}{4--1}$$

$$\frac{y-5}{-7} = \frac{x+1}{5}$$

$$5(y-5) = -7(x+1)$$

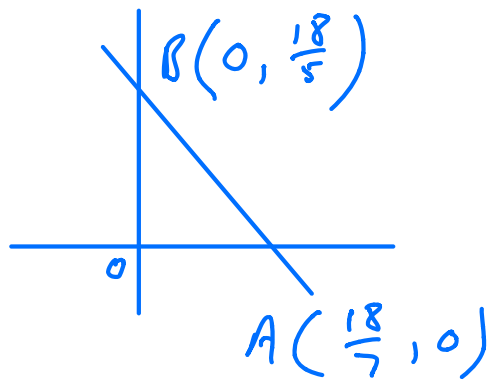
$$5y-25 = -7x-7$$

$$\underline{7x+5y-18=0}$$

$$7x+5y=18$$

$x=0 \Rightarrow y=\frac{18}{5}$

$y=0 \Rightarrow x=\frac{18}{7}$



Area of $\triangle AOB = \frac{1}{2} \times \frac{18}{7} \times \frac{18}{5}$

$= \frac{162}{35} = 4.63$ units^2

Completing the Square

Consider $(x+a)^2$

$$= x^2 + ax + ax + a^2$$

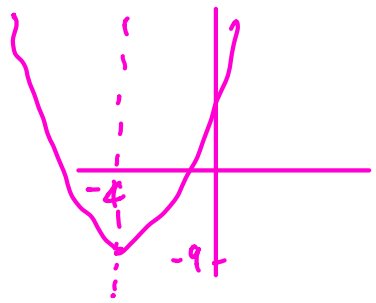
$$= x^2 + 2ax + a^2$$

Now consider

$$x^2 + 8x + 7$$

$$= (x+4)^2 + 7 - 16$$

$$= (x+4)^2 - 9$$



$(x+a)(x+4)$

$= x^2 + 8x + 16$

Ex 2

$$x^2 - 3x + 8$$

$$= \left(x - \tfrac{3}{2}\right)^2 + 8 - \frac{9}{4}$$

$$= \left(x - \tfrac{3}{2}\right)^2 + \frac{23}{4}$$

$\left(x - \frac{3}{2}\right)\left(x - \frac{3}{2}\right)$

$= x^2 - 3x + \frac{9}{4}$

Ex 3

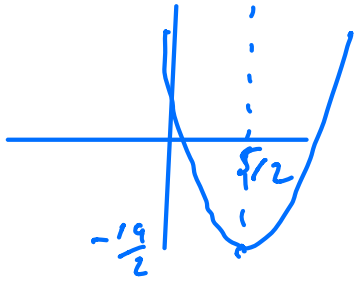
$$2x^2 - 10x + 3$$

$$2\left[x^2 - 5x + \tfrac{3}{2}\right]$$

$$2\left[\left(x - \tfrac{5}{2}\right)^2 + \tfrac{3}{2} - \tfrac{25}{4}\right]$$

$$2\left(x-\frac{5}{2}\right)^2+3-\frac{25}{2}$$

$$2\left(x-\frac{5}{2}\right)^2-\frac{19}{2}$$



The Circle

From GCSE $\quad x^2+y^2=r^2$

Circle centre $(0,0)$ radius r

$$(x-1)^2+(y+2)^2=r^2$$

Circle centre $(1,-2)$ radius r

In general,

$$(x-a)^2+(y-b)^2=r^2$$

Circle centre (a,b) radius r

Identify centre and radius

1) $(x-3)^2+(y+4)^2=36$

centre $(3,-4)$ radius 6

2) $(x-4)^2+y^2=45$

centre $(4,0)$ radius $=\sqrt{45}$ or $3\sqrt{5}$

Now consider

$$(x-3)^2 + (y+4)^2 = 36$$

$$x^2 - 6x + 9 + y^2 + 8y + 16 = 36$$

$$\underline{x^2 + y^2 - 6x + 8y - 11 = 0}$$

This circle centre $(3,-4)$ radius 6 written in a different form

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Example 1

$$x^2 + y^2 - 10x + 6y - 10 = 0$$

Find centre and radius

$$x^2 - 10x + y^2 + 6y - 10 = 0$$

$$(x-5)^2 - 25 + (y+3)^2 - 9 - 10 = 0$$

$$(x-5)^2 + (y+3)^2 = \sqrt{44}^2$$

Centre $(5,-3)$ radius $= \sqrt{44}$
