Increasing and Decreasing Functions

A function can be increasing or decreasing throughout its entire domain or for a part of its domain

To show a function is increasing throughout an interval it is necessary and sufficient to show its gradient function is positive throughout that interval

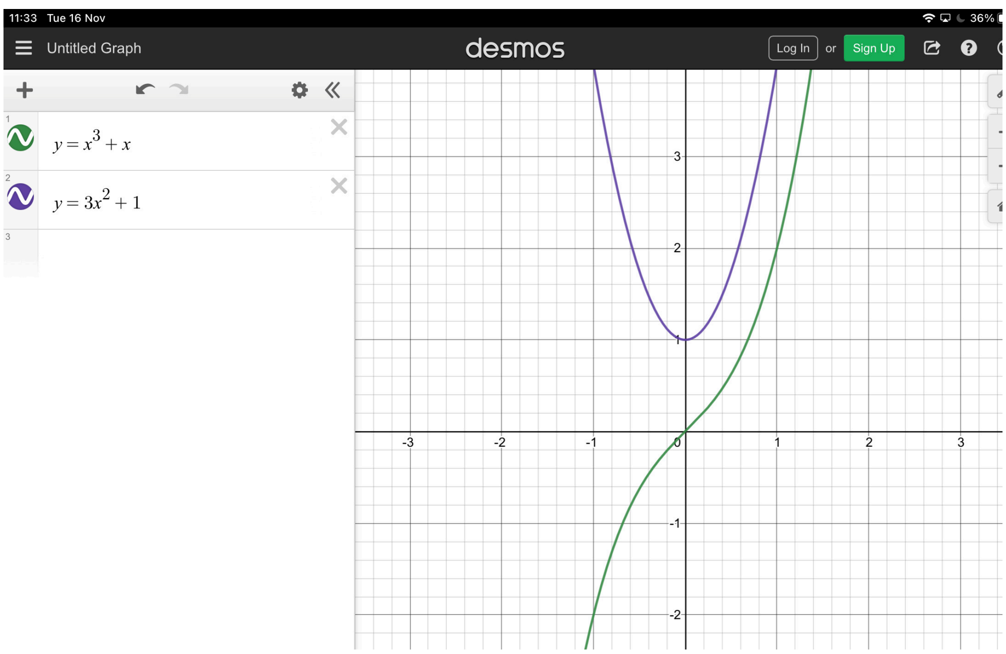
Ex 1 $\quad f(x) = x^3 + x$

Show that $f(x)$ is an increasing function for $\{x : x \in \mathbb{R}\}$

$$f'(x) = 3x^2 + 1 \geq 1 \quad \text{for all } x \in \mathbb{R}$$

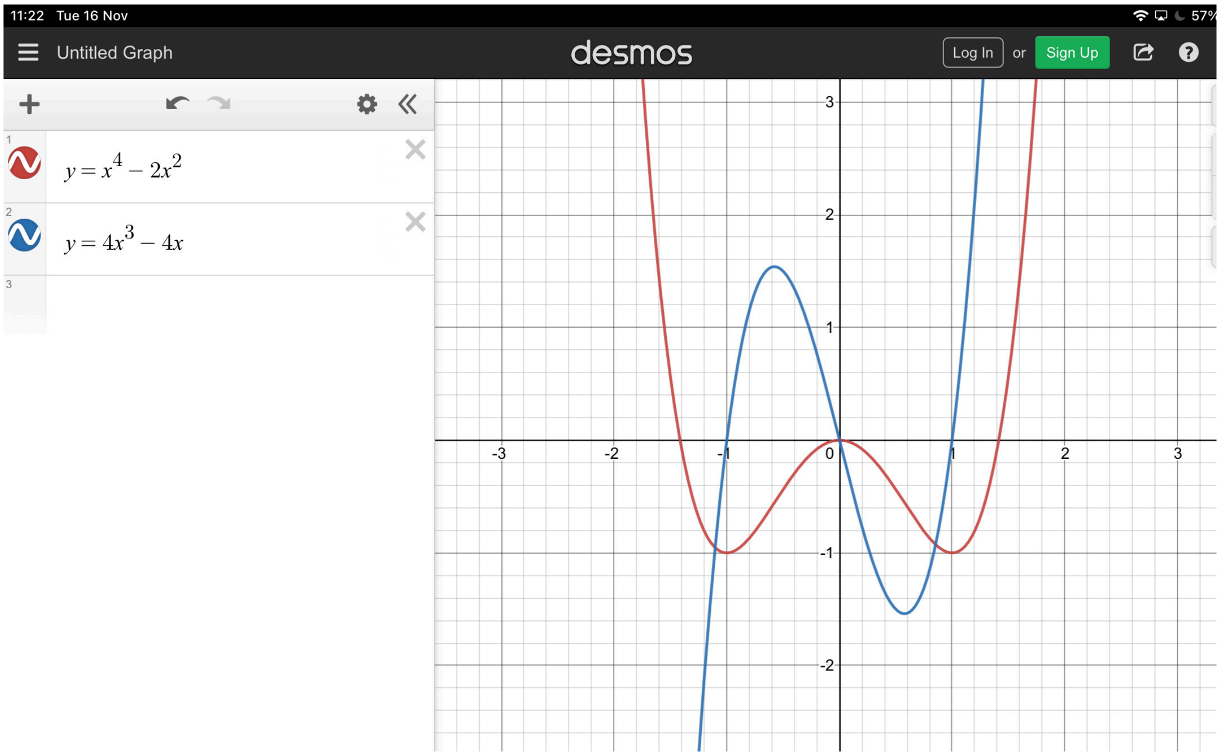
Since gradient function > 0 for all $x \in \mathbb{R}$

$f(x)$ is an increasing function throughout $\mathbb{R}$



Ex 2 $\quad f(x) = x^4 - 2x^2$

Describe when this function is increasing, decreasing or stationary.



$f(x)$ is decreasing for $x < -1$

$f(x)$ is stationary at $x = -1$

$f(x)$ is increasing for $-1 < x < 0$

$f(x)$ is stationary at $x = 0$

$f(x)$ is decreasing for $0 < x < 1$

$f(x)$ is stationary at $x = 1$

$f(x)$ is increasing for $x > 1$

Notice the value of $f'(x)$ at these points and in these intervals

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1 a) Find when $f(x)$ is increasing

$$f(x) = 3x^2 + 8x + 2$$

$$f'(x) = 6x + 8$$

$f(x)$ is increasing when $f'(x) > 0$

$$\Rightarrow \quad 6x + 8 > 0$$

$$\Rightarrow \quad 6x > -8$$

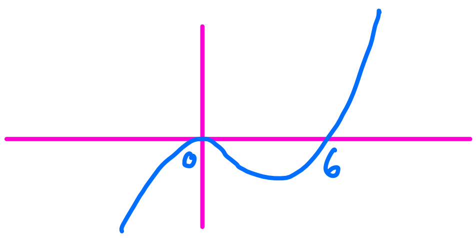
$$\Rightarrow \quad x > -\frac{4}{3}$$

1h)

$$g(x) = x^4 - 8x^3$$

$$g'(x) = 4x^3 - 24x^2$$

$$g'(x) = 4x^2(x - 6)$$



$g(x)$ is increasing for $x > 6$

Sketching Gradient Functions

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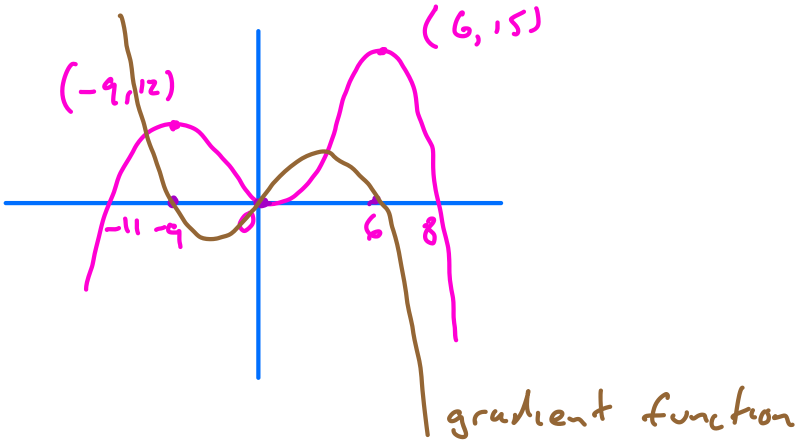
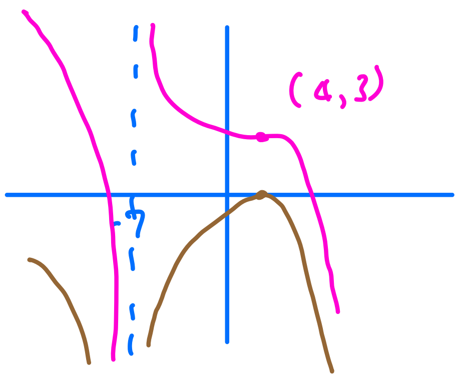
1 a)



1 c)



Homework

Exercise 12G Page 271

1b, 1f, 2b, 2f

Exercise 12 J Page 278

1b, 1d, 1e, 1f,