

8. (a) Show that the equation

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

can be written as

$$4 \cos^2 x - 9 \cos x + 2 = 0.$$

(2)

- (b) Hence solve, for  $0^\circ \leq x < 720^\circ$ ,

$$4 \sin^2 x + 9 \cos x - 6 = 0,$$

giving your answers to 1 decimal place.

(6)

a)

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

$$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$$

$$4 - 4 \cos^2 x + 9 \cos x - 6 = 0$$

$$0 = 4 \cos^2 x - 9 \cos x + 2$$


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b)

$$4 \cos^2 x - 9 \cos x + 2 = 0$$

$$4x^2$$

$$= 8$$

$$-1 - \pi$$

$$4 \cos^2 x - 9 \cos x - 8 \cos x + 2 = 0$$

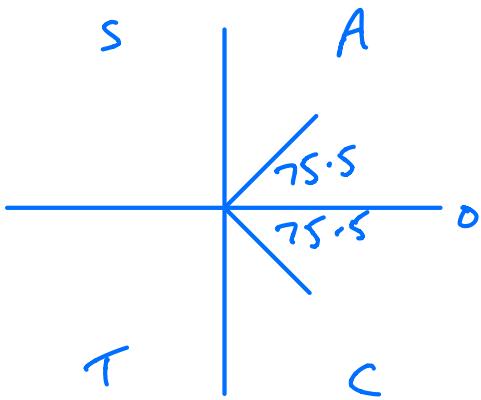
$$\cos x (4 \cos x - 1) - 2 (4 \cos x - 1) = 0$$

$$(\cos x - 2)(4 \cos x - 1) = 0$$

$$\Rightarrow 4 \cos x - 1 = 0$$

$$4 \cos x = 1$$

$$\cos x = \frac{1}{4}$$



$$\cos^{-1} \frac{1}{4} = 75.5^\circ$$

$$x = 75.5^\circ, 284.5^\circ$$

$$435.5^\circ, 644.5^\circ$$

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7. (i) Solve, for  $-180^\circ \leq \theta < 180^\circ$ ,

$$(1 + \tan \theta)(5 \sin \theta - 2) = 0.$$

(4)

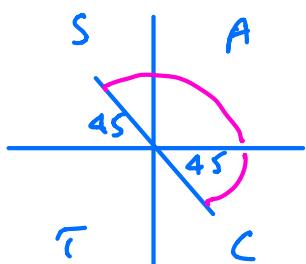
(ii) Solve, for  $0 \leq x < 360^\circ$ ,

$$4 \sin x = 3 \tan x.$$

(6)

i) Either  $1 + \tan \theta = 0$   
 $\tan \theta = -1$

$$\tan^{-1} 1 = 45^\circ$$



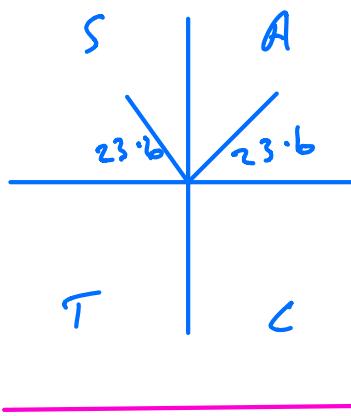
$$\theta = 135^\circ$$

$$\theta = -45^\circ$$

or  $5 \sin \theta - 2 = 0$

$$5 \sin \theta = 2$$

$$\sin \theta = \frac{2}{5}$$



$$\sin^{-1} \frac{3}{5} = 23.6^\circ$$

$$\alpha = 23.6^\circ$$

$$\alpha = 156.4^\circ$$

ii)

$$4 \sin x = 3 \tan x$$

$$4 \sin x = 3 \frac{\sin x}{\cos x}$$

$$4 \sin x \cos x - 3 \sin x = 0$$

$$\sin x (4 \cos x - 3) = 0$$

Either  $\sin x = 0 \Rightarrow x = 0^\circ$   
 $x = 180^\circ$

or  $4 \cos x - 3 = 0$

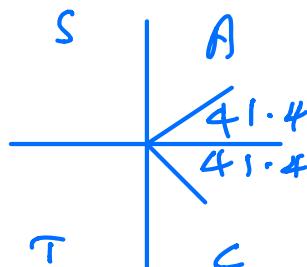
$$4 \cos x = 3$$

$$\cos x = \frac{3}{4}$$

$$\cos^{-1} \frac{3}{4} = 41.4^\circ$$

$$x = 41.4^\circ$$

$$x = 318.6^\circ$$



2. (a) Show that the equation

$$5 \sin x = 1 + 2 \cos^2 x$$

can be written in the form

$$2 \sin^2 x + 5 \sin x - 3 = 0 \quad (2)$$

- (b) Solve, for  $0^\circ \leq x < 360^\circ$ ,

$$2 \sin^2 x + 5 \sin x - 3 = 0 \quad (4)$$

### TRIGONOMETRIC EQUATIONS

(5)

a)  $5 \sin x = 1 + 2 \cos^2 x$

$$5 \sin x = 1 + 2(1 - \sin^2 x)$$

$$5 \sin x = 1 + 2 - 2 \sin^2 x$$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

b)  $2 \sin^2 x + 5 \sin x - 3 = 0$

Solve for  $0^\circ \leq x < 360^\circ$

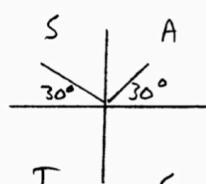
$$(2 \sin x - 1)(\sin x + 3) = 0$$

$$\Rightarrow 2 \sin x - 1 = 0 \quad \text{since } \sin x \neq -3$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$\sin^{-1} \frac{1}{2} = 30^\circ$$



$$x = 30^\circ, 150^\circ$$

# Problem Solving

6.

Figure 1

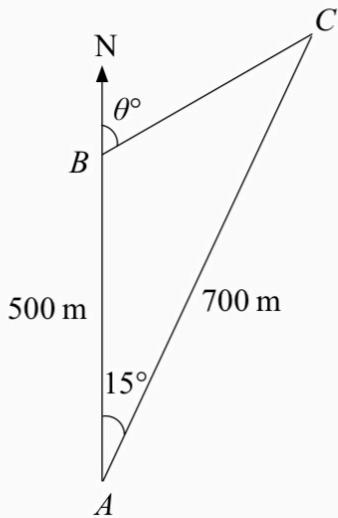


Figure 1 shows 3 yachts  $A$ ,  $B$  and  $C$  which are assumed to be in the same horizontal plane. Yacht  $B$  is 500 m due north of yacht  $A$  and yacht  $C$  is 700 m from  $A$ . The bearing of  $C$  from  $A$  is  $015^\circ$ .

- (a) Calculate the distance between yacht  $B$  and yacht  $C$ , in metres to 3 significant figures.

(3)

The bearing of yacht  $C$  from yacht  $B$  is  $\theta^\circ$ , as shown in Figure 1.

- (b) Calculate the value of  $\theta$ .

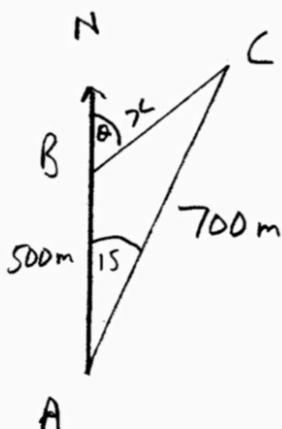
(4)

(1)

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6)

a)

Let  $BC = x$ 

Cosine rule

$$x^2 = 500^2 + 700^2 - 2 \times 500 \times 700 \cos 15^\circ$$

$$x^2 = 63852$$

$$x = 252.689$$

$$x = 253 \text{ m} \quad \text{to 3 s.f.}$$


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b) Find  $\theta$ First find  $\angle ABC$ 

Sine rule

$$\frac{252.689}{\sin 15^\circ} = \frac{700}{\sin(\angle ABC)}$$

$$252.689 \sin(\angle ABC) = 700 \sin 15^\circ$$

$$\sin(\angle ABC) = \frac{700 \sin 15^\circ}{252.689}$$

$$\angle ABC = \sin^{-1} \left( \frac{700 \sin 15^\circ}{252.689} \right)$$

$$\angle ABC = 45.8^\circ \text{ or } 134.2^\circ$$

In this case  $\angle ABC$  is obtuse  $134.2^\circ$ 

$$\theta = 180^\circ - 134.2^\circ$$

$$\theta = 45.8^\circ$$


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