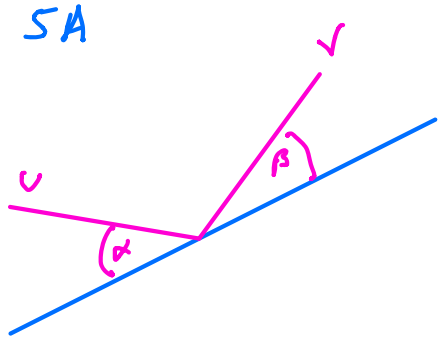
Oblique Impact

Exercise 5A

1)



$e = \frac{1}{3}$ $\tan\alpha = \frac{3}{4}$

$U\cos\alpha = V\cos\beta$ ‖ to wall

$eU\sin\alpha = V\sin\beta$

$e\tan\alpha = \tan\beta$

$\frac{1}{3} \times \frac{3}{4} = \tan\beta$

$\tan^{-1}\frac{1}{4} = \beta$

$\beta = 14.04^\circ$

$\alpha = \tan^{-1}\frac{3}{4} = 36.87^\circ$

b) Angle of deflection $= \alpha + \beta = 50.91$
$= 50.9^\circ$

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a) ‖ to wall speed component $= U\cos\alpha$

⊥ to wall speed component $= \frac{1}{3}U\sin\alpha$

$$\text{Speed} = \sqrt{U^2\cos^2\alpha + \frac{1}{9}U^2\sin^2\alpha}$$

$$= U\sqrt{\cos^2 36.87^\circ + \frac{1}{9}\sin^2 36.87}$$

$$= 0.824U$$

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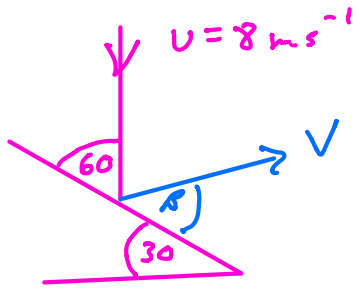
5)

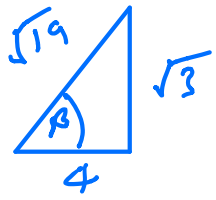


$e = \frac{1}{4}$

‖ to plane

$8\cos 60 = V\cos\beta$

$4 = V\cos\beta$



$$\tan\beta = e\tan\alpha$$
$$= \frac{1}{4}\tan 60$$
$$= \frac{\sqrt{3}}{4}$$

$$\frac{4}{\cos\beta} = V$$

$$\frac{4}{\frac{4}{\sqrt{19}}} = V$$

$$V = \sqrt{19} \ \text{ms}^{-1}$$

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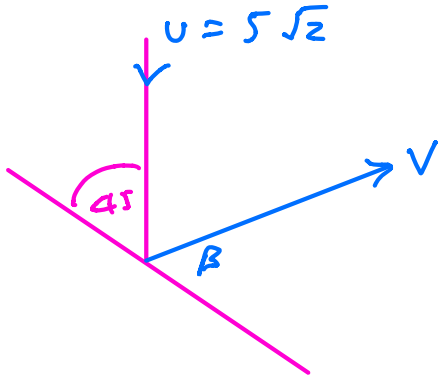
7)



$$e = \frac{1}{2}$$

a)

$$V = \sqrt{(5\sqrt{2}\cos 45^\circ)^2 + \left(\tfrac{1}{2}5\sqrt{2}\sin 45\right)^2}$$

$$V = \frac{5\sqrt{5}}{2} = 5.59 \ \text{ms}^{-1}$$

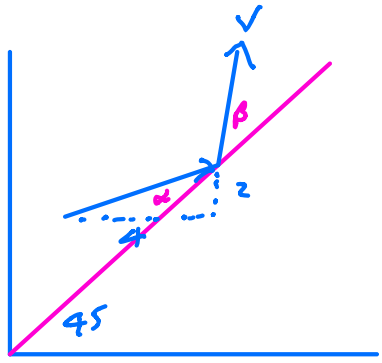
b) $\perp$ to plane change in momentum = impulse

$$m\left(5\sqrt{2}\sin 45 + e\,5\sqrt{2}\sin 45\right)$$

$$m\left(5 + \tfrac{5}{2}\right) = 0.75 \times \tfrac{15}{2}$$

$$= 5.625 \ \text{kg ms}^{-1}$$

11)



$$e = \frac{1}{3}$$

$$u = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}||\underline{b}|}$$

$$\alpha = \cos^{-1}\left(\frac{\begin{pmatrix} 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 4 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right|} \right)$$

$$\alpha = \cos^{-1}\left(\frac{6}{\sqrt{20}\sqrt{2}} \right)$$

$$\underline{\alpha = 18.435^\circ}$$

$$\tan\beta = e\tan\alpha$$

$$\beta = \tan^{-1}\left(\frac{1}{3}\tan 18.435^\circ \right)$$

$$\underline{\beta = 6.340^\circ}$$

$$|u| = \sqrt{4^2 + 2^2} = \sqrt{20}$$

$$|v| = \sqrt{(u\cos\alpha)^2 + (eu\sin\alpha)^2}$$

$$= \sqrt{20\cos^2 18.435 + \frac{1}{9} \cdot 20 \cdot \sin^2 18.435}$$

$$|v| = 4.3587 \text{ ms}^{-1}$$

$$\underline{v} = 4.3587\cos(45 + 6.34)\underline{i} + 4.3587\sin(51.34)\underline{j}$$

$$= 2.72\,\underline{i} + 3.40\,\underline{j}$$