OCR MEI MECH 2 JUN 05

2 A car of mass 850 kg is travelling along a road that is straight but not level.

On one section of the road the car travels at constant speed and gains a vertical height of 60 m in 20 seconds. Non-gravitational resistances to its motion (e.g. air resistance) are negligible.

(i) Show that the average power produced by the car is about 25 kW. [2]

On a *horizontal* section of the road, the car develops a constant power of exactly 25 kW and there is a constant resistance of 800 N to its motion.

(ii) Calculate the maximum possible steady speed of the car. [3]

(iii) Find the driving force and acceleration of the car when its speed is $10\,\mathrm{m\,s^{-1}}$. [3]

When travelling along the horizontal section of road, the car accelerates from $15\,\mathrm{m\,s^{-1}}$ to $20\,\mathrm{m\,s^{-1}}$ in 6.90 seconds with the same constant power and constant resistance.

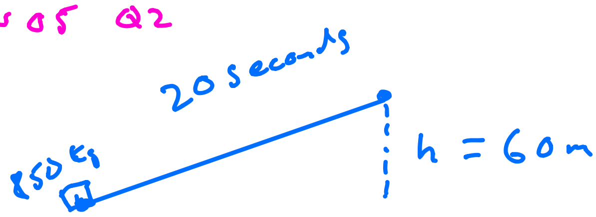
(iv) By considering work and energy, find how far the car travels while it is accelerating. [6]

When the car is travelling at $20\,\mathrm{m\,s^{-1}}$ up a constant slope inclined at arcsin (0.05) to the horizontal, the driving force is removed. Subsequently, the resistance to the motion of the car remains constant at 800 N.

(v) What is the speed of the car when it has travelled a further 105 m up the slope? [5]

Jun 05 Q2

i)



KE constant

increase in GPE = mgh

$850 \times 9.8 \times 60$

$= 499{,}800 \text{ J}$

Car produces 499,800 J in 20 s

$\therefore$ Power $= \dfrac{499{,}800}{20} = 24{,}990 \text{ W}$

$\approx 25 \text{ kW}$

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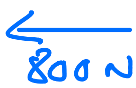
ii)

Power = 25000 W



At top speed $F = 800\text{N}$ opposing resistance

so $Fv = \text{Power}$

$800v = 25000$

$v = \dfrac{25000}{800} = 31.25 \text{ m s}^{-1}$

$= 31\tfrac{1}{4} \text{ m s}^{-1}$

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iii)

$Fv = \text{Power}$

$F \times 10 = 25000$

$\underline{F = 2500 \text{ N}}$ Driving Force

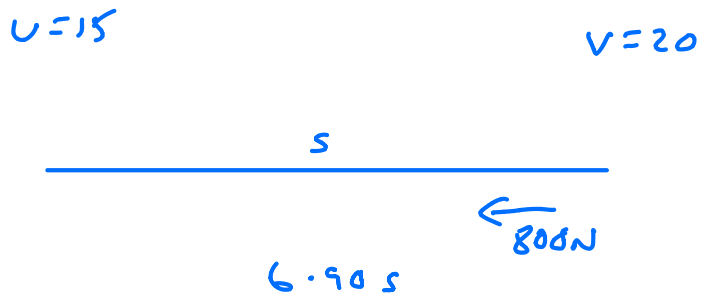
Resultant Force $F' = 2500 - 800 = 1700 \text{ N}$

$$F' = ma$$

$$1700 = 850a$$

$$\underline{a = 2\,ms^{-2}}$$

iv)



Energy produced by car $= 25000 \times 6.90$ J

$= 172500$ J

172500 J $=$ Increase in KE $+$ Work Against Resistance

$$= \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2 + 800s$$

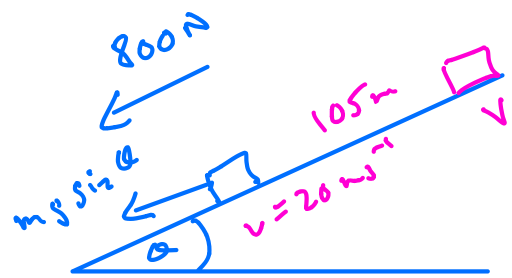
$$= \tfrac{1}{2} \times 850(20^2 - 15^2) + 800s$$

$$\frac{172500 - 74375}{800} = s$$

$$s = 122.656$$

$$\underline{s = 123\,m}$$

v)



$\theta = \sin^{-1} 0.05$

Loss in KE = Increase in GPE + Work Against Resistance

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh + 800 \times 105$$

$$\frac{1}{2} \times 850(20^2 - v^2) = 850 \times 9.8 \times 105 \times 0.05 + 800 \times 105$$

$$425(400 - v^2) = 127732.5$$

$$400 - v^2 = \frac{127732.5}{425}$$

$$400 - \frac{51093}{170} = v^2$$

$$v = 9.97 \text{ ms}^{-1}$$

Jan 06

4 A block of mass 20 kg is pulled by a light, horizontal string over a rough, horizontal plane. During 6 seconds, the work done against resistances is 510 J and the speed of the block increases from $5\,\text{m s}^{-1}$ to $8\,\text{m s}^{-1}$.

(i) Calculate the power of the pulling force. [4]

The block is now put on a rough plane that is at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. The frictional resistance to sliding is $11g$ N. A light string parallel to the plane is connected to the block. The string passes over a smooth pulley and is connected to a freely hanging sphere of mass m kg, as shown in Fig. 4.

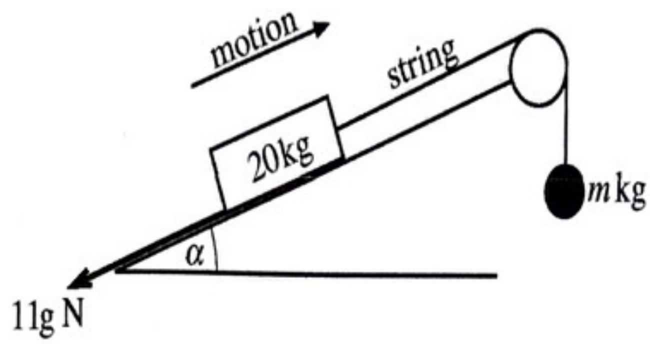


Fig. 4

In parts **(ii)** and **(iii)**, the sphere is pulled downwards and then released when travelling at a speed of $4\,\text{m s}^{-1}$ vertically downwards. The block never reaches the pulley.

(ii) Suppose that $m = 5$ and that after the sphere is released the block moves x m up the plane before coming to rest.

(*A*) Find an expression in terms of x for the change in gravitational potential energy of the system, stating whether this is a gain or a loss. [4]

(*B*) Find an expression in terms of x for the work done against friction. [1]

(*C*) Making use of your answers to parts (*A*) and (*B*), find the value of x. [3]

(iii) Suppose instead that $m = 15$. Calculate the speed of the sphere when it has fallen a distance 0.5 m from its point of release. [4]

Jan 06 Q4

i) 6 seconds

Work Done = Increase in KE + Work Against Resistance

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 + 510\,J$

$= \frac{1}{2} \times 20(8^2 - 5^2) + 510$

$= 900\,J$

Power $= \frac{900}{6} = 150\,W$

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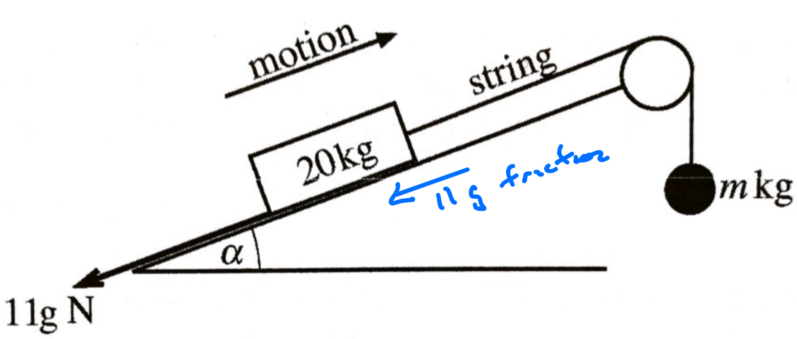
ii)



$\sin\alpha = \frac{3}{5} = 0.6$

Block Ball

A) Change in GPE $= 20 \times 9.8 \times x \sin\alpha - mgx$

$= 20 \times 9.8 \times x \times 0.6 - 5 \times 9.8x$

$= 117.6x - 49x$

$= 68.6x\,J$ (a gain)

B) Work against friction $= 11gx$

c) Loss in KE = Gain in GPE + Work Against Friction

$$\frac{1}{2} \times 25 \times 4^2 = 68.6x + 11 \times 9.8x$$

$$200 = 176.4x$$

$$x = \frac{200}{176.4}$$

$$\underline{x = 1.13 \text{ m}}$$

iii) Change in GPE of System

Block — Ball

mgh

$$20 \times 9.8 \times 0.5 \times 0.6 - 15 \times 9.8 \times 0.5$$

$$= -14.7 \text{ J} \quad \text{(loss)}$$

Loss in GPE + Loss in KE = Work against friction

$$14.7 + \frac{1}{2}m(u^2 - v^2) = 11g \times 0.5$$

$$14.7 + \frac{1}{2} \times 35(4^2 - v^2) = 11 \times 9.8 \times 0.5$$

$$17.5(16 - v^2) = 11 \times 9.8 \times 0.5 - 14.7$$

$$16 - v^2 = \frac{11 \times 9.8 \times 0.5 - 14.7}{17.5}$$

$$\sqrt{16 - \left(\frac{11 \times 9.8 \times 0.5 - 14.7}{17.5}\right)} = V$$

$V = 3.71 \text{ ms}^{-1}$

Jun 06

3 **(a)** A car of mass 900 kg is travelling at a steady speed of 16 m s^{-1} up a hill inclined at arcsin 0.1 to the horizontal. The power required to do this is 20 kW.

Calculate the resistance to the motion of the car. [4]

(b) A small box of mass 11 kg is placed on a uniform rough slope inclined at $\arccos \frac{12}{13}$ to the horizontal. The coefficient of friction between the box and the slope is μ.

(i) Show that if the box stays at rest then $\mu \geqslant \frac{5}{12}$. [3]

For the remainder of this question, the box moves on a part of the slope where $\mu = 0.2$.

The box is projected up the slope from a point P with an initial speed of $v \text{ m s}^{-1}$. It travels a distance of 1.5 m along the slope before coming instantaneously to rest. During this motion, the work done against air resistance is 6 joules per metre.

(ii) Calculate the value of v. [5]

As the box slides back down the slope, it passes through its point of projection P and later reaches its initial speed at a point Q. During this motion, once again the work done against air resistance is 6 joules per metre.

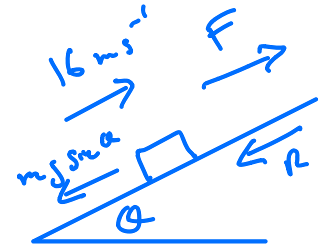
(iii) Calculate the distance PQ. [6]

Jun 06

3)

$\sin\theta = 0.1$

a)



Power = Fv

20000 = F × 16

$F = \frac{20000}{16} = 1250\,N$

Steady speed so no acceleration
so no resultant force

$1250 = mg\sin\theta + R$

$1250 - 900 \times 9.8 \times 0.1 = R$

$R = 368\,N$

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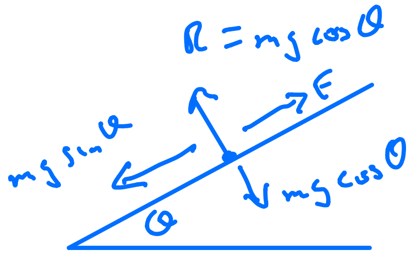
b) Limiting friction = μR

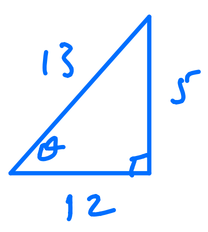


If at rest

$\mu R \geqslant mg\sin\theta$

$\mu mg\cos\theta \geqslant mg\sin\theta$

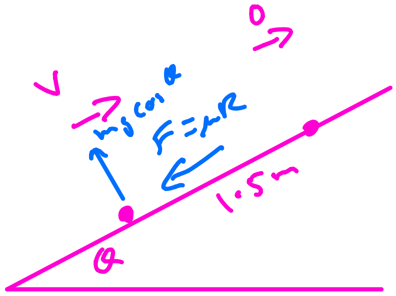
$\mu \geqslant \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$



$\tan\theta = \frac{5}{12}$

$\mu \geqslant \frac{5}{12}$

ii)



$\cos\theta = \frac{12}{13}$

$\sin\theta = \frac{5}{13}$

$\mu = 0.2$

Loss in KE = Increase in GPE + Work against Resistances (air, friction)

$$\frac{1}{2}mv^2 = mg \times 1.5 \times \frac{5}{13} + 6 \times 1.5 + 6 \times 0.2 \times mg \times \frac{12}{13}$$

$$\frac{1}{2} \times 11 v^2 = 11 \times 9.8 \times 1.5 \times \frac{5}{13} + 9 + 1.5 \times 0.2 \times 11 \times 9.8 \times \frac{12}{13}$$

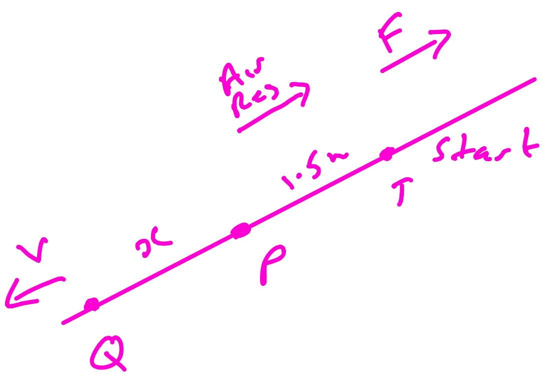
$$5.5 v^2 = 101.04$$

$$v = \sqrt{\frac{101.04}{5.5}}$$

$v = 4.28622774($

$$\underline{v = 4.29 \text{ ms}^{-1}}$$

iii)



Let $PQ = x$

Loss in GPE = Gain in KE + Work Friction + Work Air

$$mgh = \frac{1}{2}mv^2 + \mu R(x+1.5) + 6(x+1.5)$$

$$11 \times 9.8 \times (x+1.5) \times \frac{5}{13} = \frac{11}{2}v^2 + 0.2 \times 11 \times 9.8 \times \frac{12}{13}(x+1.5) + 6(x+1.5)$$

$$\frac{539}{13}(x+1.5) = 101.0446 + \frac{8418}{325}(x+1.5)$$

$$(x+1.5) = \frac{101.0446}{\left(\frac{539}{13} - \frac{8418}{325}\right)}$$

$$x + 1.5 = 6.494$$

$$x = 4.99\ m$$