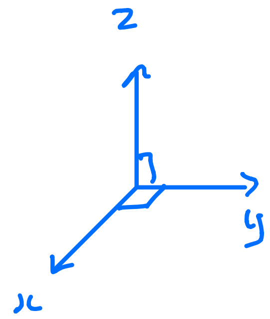
Vectors



$\times A(2, 3, 1)$

A has position vector $2\underline{i} + 3\underline{j} + \underline{k}$

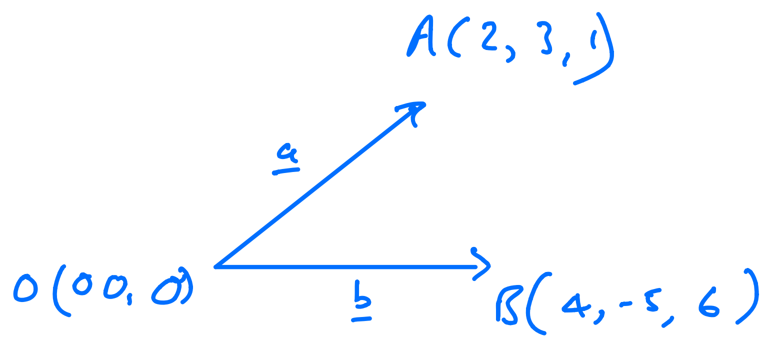
or $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

where $\underline{i}$ is a unit vector in x-direction
$\underline{j}$ —— " —— y-direction
$\underline{k}$ —— " —— z-direction

The position vector of a point A is the vector from the origin (0,0,0) to A(2, 3, 1)

Finding vectors between points

A(2, 3, 1)
B(4 -5, 6)
C(0, 7, -2)



$\underline{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix}$

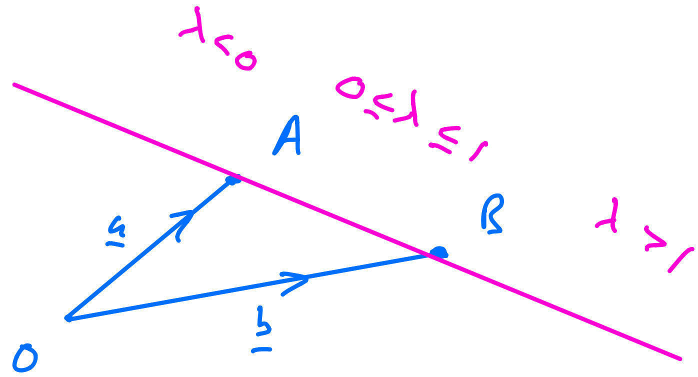
$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{a} + \underline{b} \qquad = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ -5 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \\ 5 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -8 \\ 5 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} -4 \\ 12 \\ -8 \end{pmatrix}$ $\qquad$ $\overrightarrow{AC} = \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$

Vector Equation of a Line through A with position vector $\underline{a}$ and B with position vector $\underline{b}$



$$\underline{r} = \overrightarrow{OA} + \lambda \overrightarrow{AB}$$

$$\underline{r} = \underline{a} + \lambda(\underline{b} - \underline{a})$$

$$\underline{r} = (1-\lambda)\underline{a} + \lambda \underline{b}$$

Vector equns of lines are not unique

We need any point on the line and a direction vector to define the line

Ex 1 Find vector eqn of line through A(2, 4, 5) and B(6, 2, 3)

$$\underline{r} = \overrightarrow{OA} + \lambda \overrightarrow{AB}$$

$$\underline{r} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$$

or $$\underline{r} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

or $$\underline{r} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

All these represent the same line but λ would take different values in each equation to represent a specific point

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Cartesian Form of a Line in Space

Suppose $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{r} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$x - a_1 = \lambda v_1$$

$$y - a_2 = \lambda v_2$$

$$z - a_3 = \lambda v_3$$

$$\lambda = \frac{x - a_1}{v_1} = \frac{y - a_2}{v_2} = \frac{z - a_3}{v_3}$$

There are two special cases when one or two of u_1, u_2, u_3 are equal to zero

Examples

1) $\underline{r} = \begin{pmatrix} 2 \\ 5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 8 \\ 1 \end{pmatrix}$

$$\frac{x-2}{3} = \frac{y-5}{8} = \frac{z-7}{1}$$

2) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{r} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$

$$\cancel{\frac{x-3}{0}} = \frac{y-0}{-2} = \frac{z-4}{3}$$

$x = 3$

Cartesian Eqn $\quad \frac{y-0}{-2} = \frac{z-4}{3}$ and $x = 3$

3) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{r} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$

$$\cancel{\frac{x-2}{0}} = \cancel{\frac{y-3}{0}} = \frac{z-4}{5}$$

Cartesian Eqn $x = 2$ and $y = 3$

Converting Cartesian form to Vector form

Ex 1 $\quad \dfrac{x-5}{2} = \dfrac{y+3}{1} = \dfrac{z}{7}$

$$\underline{r} = \begin{pmatrix} 5 \\ -3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$$

Ex 2 $\quad \dfrac{y-3}{2} = \dfrac{z+4}{5}$ and $x = 7$

$$\underline{r} = \begin{pmatrix} 7 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix}$$

Ex 3 $\quad x = 2$ and $y = 3$

$$\underline{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

could be any number

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