U61M Vectors 2D SUVAT Mark Scheme

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| **2(a)** | $\mathbf{v} = 4\mathbf{i} + (-3\mathbf{i} + 12\mathbf{j})t$ | M1<br>A1 | 2 | use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ |
| **(b)** | $t = 0.5,\ \ \mathbf{v} = 2.5\mathbf{i} + 6\mathbf{j}$ | B1√ | | √ 2 terms and t subs |
| | Speed $= \sqrt{(2.5^2 + 6^2)}$ | M1 | | 2 terms |
| | Speed $= 6.5\,\mathrm{m\,s^{-1}}$ | A1√ | 3 | √ 2 terms |
| | **Total** | | **5** | |

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| 6 | $-6\mathbf{i} + \mathbf{j} = \mathbf{u} + 3(2\mathbf{i} - 5\mathbf{j})$<br>$\Rightarrow \mathbf{u} = -12\mathbf{i} + 16\mathbf{j}$<br>$\Rightarrow u = \sqrt{(-12)^2 + 16^2} = 20$ | M1 A1<br>A1 cso<br>M1 A1<br>**[5]** |

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| **6(a)** | $\mathbf{d} = 3\mathbf{i} - 6\mathbf{j}$ | B1 | | Accept $\pm\mathbf{d}$ or displacements of 3, 6 shown on a diagram |
| | $3\mathbf{i} - 6\mathbf{j} = (\mathbf{i} - 2\mathbf{j})t$ | M1 | | Or equivalent method for $t$<br>Accept ratio of vectors leading directly to $\pm 3$ |
| | $t = 3$ | A1 | 3 | CAO |
| **(b)(i)** | $\mathbf{r} = (\mathbf{i} - 2\mathbf{j}) \times 4 + \frac{1}{2} \times 2\mathbf{j} \times 16$ | M1 | | Full method for vector expression giving change in position |
| | | A1 | | For correct subs<br>(gives $4\mathbf{i} + 8\mathbf{j}$) |
| | $+6\mathbf{i} - 4\mathbf{j}$ | M1 | | |
| | $= 10\mathbf{i} + 4\mathbf{j}$ | A1F | 4 | FT slip provided obtain vector expression<br>$(\mathbf{u} = 0$ gives $6\mathbf{i} + 12\mathbf{j})$ |
| **(ii)** | $A(3,2) \quad C(10,4)$<br>$\mathbf{d} = 7\mathbf{i} + 2\mathbf{j}$ | M1 | | Attempt to find vector $\overline{AC}$ or $\overline{CA}$ (using candidate's $C$ |
| | $\lvert\mathbf{d}\rvert = \sqrt{7^2 + 2^2}$<br>$AC = \sqrt{53} = 7.28$ | A1F | 2 | FT **d** provided two non-zero components<br>Accept $\sqrt{53}$ |
| | **Total** | | **9** | |

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| **8(a)** | $75\mathbf{i} = (5\mathbf{i} - 2\mathbf{j}) \times 10 + \frac{1}{2}\mathbf{a} \times 10^2$ | M1 | | Equation to find **a** from $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ |
| | | A1 | | Correct expression |
| | $\mathbf{a} = \frac{75\mathbf{i} - 50\mathbf{i} + 20\mathbf{j}}{50} = 0.5\mathbf{i} + 0.4\mathbf{j}$ | A1 | 3 | **AG** Correct **a** from correct working |
| **(b)** | $\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 8 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 8^2$ | M1 | | Expression for **r** using $t = 8$ with no extra terms |
| | | A1 | | Correct expressions |
| | $= 56\mathbf{i} - 3.2\mathbf{j}$ | A1 | 3 | Correct position vector |
| **(c)** | $\mathbf{v} = (5 + 0.5t)\mathbf{i} + (0.4t - 2)\mathbf{j}$ | M1A1 | | Expression for **v**. Correct expression |
| | $0.4t - 2 = 0$ | dM1 | | **j** component equal to zero |
| | $t = \frac{2}{0.4} = 5$ | A1 | | Correct $t$ |
| | $\mathbf{r} = (5\mathbf{i} - 2\mathbf{j}) \times 5 + \frac{1}{2}(0.5\mathbf{i} + 0.4\mathbf{j}) \times 5^2$ | dM1 | | Expression for **r** using $t$ from **j** component equal to zero |
| | $= 31.25\mathbf{i} - 5\mathbf{j}$<br>$= 31.3\mathbf{i} - 5\mathbf{j}$ | A1 | 6 | Correct position vector |
| | **Total** | | **12** | |

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| **8(a)** | $\mathbf{u} = 5\mathbf{i}$ or $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ | B1 | 1 | Correct velocity |
| **(b)** | $\mathbf{v} = 5\mathbf{i} + (-0.2\mathbf{i} + 0.25\mathbf{j})t$ | M1 | | Use of constant acceleration equation, with **u** and **a** not zero |
| | | A1 | 2 | Correct velocity<br>M1A0 for using 5**j** or just 5 |
| | **OR**<br>$\mathbf{v} = \begin{bmatrix} 5 - 0.2t \\ 0.25t \end{bmatrix}$ | | | |
| **(c)** | $5 - 0.2t = 0$ | M1<br>A1 | | Easterly component zero<br>Correct equation |
| | $t = \dfrac{5}{0.2} = 25$ seconds | A1 | 3 | Correct $t$ |
| **(d)** | $\mathbf{r} = 5\mathbf{i} \times 25 + \dfrac{1}{2}(-0.2\mathbf{i} + 0.25\mathbf{j}) \times 25^2$ | M1 | | Use of constant acceleration equation with $t$ from part (c) |
| | | A1F | | Correct expression based on $t$ from part (c) |
| | $= 62.5\mathbf{i} + 78.125\mathbf{j}$ | A1 | | Correct simplification CAO |
| | $\theta = \tan^{-1}\left(\dfrac{62.5}{78.125}\right)$ | dM1<br>A1F | | Using tan to find the angle<br>Correct expression based on $t$ from part (c), with correct two values(either way) |
| | $= 038.7°$ | A1 | 6 | Correct angle<br>Accept 38.6° or 039° |

| Question Number | Scheme | Marks |
|---|---|---|
| 8 | (a) Speed of ball = $\sqrt{(5^2 + 8^2)}$ $\approx$ $\underline{9.43 \text{ m s}^{-1}}$ | M1 A1 (2) |
| | (b) p.v. of ball = $(2\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 8\mathbf{j})t$ | M1 A1 (2) |
| | (c) North of $B$ when **i** components same, i.e. $2 + 5t = 10$ | M1 |
| | $t = \underline{1.6 \text{ s}}$ | A1 (2) |
| | (d) When $t = 1.6$, p.v. of ball = $10\mathbf{i} + 13.8\mathbf{j}$ (or **j** component = 13.8) | M1 A1 ↓ |
| | Distance travelled by 2nd player = 13.8 – 6 = 6.8 | M1 A1 ↓ |
| | Speed = $6.8 \div 1.6 = \underline{4.25 \text{ m s}^{-1}}$ | M1 A1 (6) |
| | *or* $[(2 + 5t)\mathbf{i} +] (1 + 8t)\mathbf{j} = [10\mathbf{i} +] (7 + vt)\mathbf{j}$ (pv's or **j** components same) | M1 A1 ↓ |
| | Using $t = 1.6$: $1 + 12.8 = 7 + 1.6v$ (equn in $v$ only) | M1 A1 ↓ |
| | $v = \underline{4.25 \text{ m s}^{-1}}$ | M1 A1 |
| | (e) Allow for friction on field (i.e. velocity of ball not constant) | B1 (1) |
| | *or* allow for vertical component of motion of ball | |
| | (a) M1 Valid attempt at speed (square, add and squ. root cpts) | |
| | (b) M1 needs non-zero p.v. + (attempt at veloc vector) x $t$. Must be vector | |
| | (d) 2nd M1 – allow if finding displacement *vector* (e.g. if using wrong time)<br>3rd M1 for getting speed as a *scalar* (and final answer must be as a scalar). But if they get e.g. '4.25**j**', allow M1 A0 | |
| | (e) Allow 'wind', 'spin', 'time for player to accelerate', size of ball<br>Do not allow on their own 'swerve', 'weight of ball'. | |

| Question Number | Scheme | Marks |
|---|---|---|
| 6. | (a) Speed of $A = \sqrt{(1^2 + 6^2)} \approx \underline{6.08 \text{ m s}^{-1}}$ | M1 A1 (2) |
| | (b)  $\tan\theta = 1/6 \Rightarrow \theta \approx 9.46°$ | M1 A1 |
| | Bearing $\approx$ <u>351</u> | A1 (3) |
| | (c) P.v. of $A$ at time $t = (2 - t)\mathbf{i} + (-10 + 6t)\mathbf{j}$ | |
| | p.v. of $B$ at time $t = (-26 + 3t)\mathbf{i} + (4 + 4t)\mathbf{j}$ | B1 (either) |
| | (E.g.) $\mathbf{i}$ components equal $\Rightarrow 2 - t = -26 + 3t \Rightarrow t = 7$ | M1 A1 |
| | $\mathbf{j}$ components at $t = 7$: $A$: $-10 + 6t = 32$ | ↓ |
| | $B$: $4 + 4t = 32$ | M1 |
| | Same, so collide at $t = 7$ s at point with p.v. $(-5\mathbf{i} + 32\mathbf{j})$ m | A1 cso (5) |
| | (d) New velocity of $B = \frac{8}{5}(3\mathbf{i} + 4\mathbf{j})$ m s$^{-1}$ | B1 |
| | P.v. of $B$ at 7 s $= -26\mathbf{i} + 4\mathbf{j} + 1.6(3\mathbf{i} + 4\mathbf{j}) \times 7 = 7.6\mathbf{i} + 48.8\mathbf{j}$ | M1 A1 |
| | $\underline{PB} = \mathbf{b} - \mathbf{p} = 12.6\mathbf{i} + 16.8\mathbf{j}$ (in numbers) | ↓ M1 |
| | Distance $= \sqrt{(12.6^2 + 16.8^2)} = \underline{21 \text{ m}}$ | ↓ M1 A1 (6) |
| | | **16** |