Exercise 4B

1) $2x^3 + 5x^2 - 2x + 3 = 0$

$\Sigma\alpha = -\frac{5}{2}$ $\quad \Sigma\alpha\beta = -\frac{2}{2} = -1$ $\quad \alpha\beta\gamma = -\frac{3}{2}$

a) $\alpha + \beta + \gamma = -\frac{5}{2}$

b) $\alpha\beta\gamma = -\frac{3}{2}$

c) $\alpha\beta + \beta\gamma + \alpha\gamma = -1$

d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{\Sigma\alpha\beta}{\alpha\beta\gamma} = \frac{-1}{-\frac{3}{2}} = \frac{2}{3}$

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6) $ax^3 + bx^2 + cx + d = 0$ $\quad \alpha = \frac{5}{4}$ $\quad \beta = -\frac{3}{2}$

$\gamma = \frac{1}{2}$

$\Sigma\alpha = \frac{1}{4}$ $\quad \Sigma\alpha\beta = \frac{5}{4}\left(-\frac{3}{2}\right) + \frac{5}{4}\cdot\frac{1}{2} + \left(-\frac{3}{2}\right)\cdot\frac{1}{2}$

$= -\frac{15}{8} + \frac{5}{8} - \frac{6}{8} = -2$

$\alpha\beta\gamma = \frac{5}{4}\left(-\frac{3}{2}\right)\frac{1}{2} = -\frac{15}{16}$

$x^3 + \frac{b}{a}x^2 + \frac{c}{a}x + \frac{d}{a} = 0$

$x^3 - \frac{1}{4}x^2 - 2x + \frac{15}{16} = 0$

$16x^3 - 4x^2 - 32x + 15 = 0$

Exercise 4C

1) $4x^4 + 3x^3 + 2x^2 - 5x - 4 = 0$

a) $\alpha + \beta + \gamma + \delta = \Sigma\alpha = -\frac{3}{4}$

b) $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \Sigma\alpha\beta = \frac{2}{4} = \frac{1}{2}$

c) $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{5}{4}$

$\alpha\beta\gamma\delta = -\frac{-4}{4} = 1$ (Aside)

d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\Sigma\alpha\beta\gamma}{\alpha\beta\gamma\delta} = \frac{\frac{5}{4}}{1} = \frac{5}{4}$

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8) $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$

roots $\alpha, \alpha+k, \alpha+2k, \alpha+3k$

$\Sigma\alpha = 16 = 4\alpha + 6k$ (1)

$\Sigma\alpha\beta = 86 = \alpha(\alpha+k+\alpha+2k+\alpha+3k)$
$+ (\alpha+k)(\alpha+2k+\alpha+3k)$
$+ (\alpha+2k)(\alpha+3k)$

$= \alpha(3\alpha+6k) + (\alpha+k)(2\alpha+5k)$
$+ (\alpha+2k)(\alpha+3k)$ (2)

From (1) $\alpha = \frac{16-6k}{4} = 4 - 1.5k$

Sub in ②

$$86 = (4 - 1.5k)(12 + 1.5k) + (4 - 0.5k)(8 + 2k) + (4 + 0.5k)(4 + 1.5k)$$

$$86 = 48 - 18k + 6k - 2.25k^2 + 32 - 4k + 8k - k^2 + 16 + 2k + 6k + 0.75k^2$$

$$86 = 96 - 2.5k^2$$

$$2.5k^2 = 96 - 86$$

$$k^2 = \frac{10}{2.5} = 4$$

$$k = \sqrt{4} = 2$$

$$\alpha = 4 - 1.5k = 4 - 3 = 1$$

$$\alpha = 1, \quad k = 2$$

Roots $1, 3, 5, 7$

MEI Exam Questions

5 The cubic equation $x^3 + 3x^2 - 7x + 1 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [2]

(ii) Find the cubic equation with roots 2α, 2β and 2γ, simplifying your answer as far as possible. [4]

i) $\alpha + \beta + \gamma = -3$

$\alpha\beta + \beta\gamma + \gamma\alpha = -7$

$\alpha\beta\gamma = -1$

ii) $\sum 2\alpha = 2\alpha + 2\beta + 2\gamma = 2(\alpha+\beta+\gamma)$

$= 2(-3)$

$= -6$

$\sum 2\alpha 2\beta = 2\alpha 2\beta + 2\alpha 2\gamma + 2\beta 2\gamma$

$= 4(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 4(-7)$

$= -28$

$2\alpha 2\beta 2\gamma = 8\alpha\beta\gamma$

$= 8(-1)$

$= -8$

$$\underline{x^3 + 6x^2 - 28x + 8 = 0}$$

Method 2 Let $w = 2x$

$\frac{w}{2} = x$

$\left(\frac{w}{2}\right)^3 + 3\left(\frac{w}{2}\right)^2 - 7\left(\frac{w}{2}\right) + 1 = 0$

$\frac{w^3}{8} + \frac{3w^2}{4} - \frac{7w}{2} + 1 = 0$

$w^3 + 6w^2 - 28w + 8 = 0$

Jan 06

4 The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [1]

(ii) Hence find the value of $\alpha^2 + \beta^2$. [2]

(iii) Find a quadratic equation which has roots 2α and 2β. [2]

Jun 05

i) $\alpha + \beta = 2$, $\alpha\beta = 4$

ii) $(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 2^2 - 2(4)$

$= -4$

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iii) $\Sigma 2\alpha = 2\alpha + 2\beta = 2(\alpha+\beta)$

$= 2(2)$

$= 4$

$2\alpha \times 2\beta = 4\alpha\beta = 4 \times 4 = 16$

Eqn is $x^2 - 4x + 16 = 0$

Hwk Exercise 4B Q2, Q4

Exercise 4C Q2, Q5