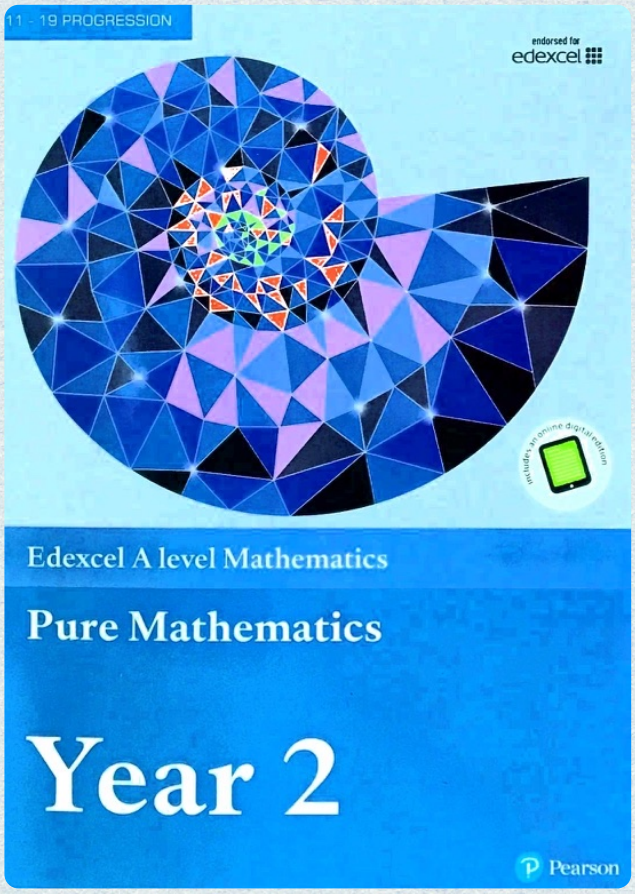

Chapter 4

Binomial Expansion

4.2 Expanding (a + bx)^n

Expanding $(a+bx)^n$

$$= \left(a\left(1+\frac{b}{a}x\right)\right)^n$$

$$= a^n\left(1+\frac{b}{a}x\right)^n$$

Expand $(2+3x)^{-2}$ as far as the term in x^3 and state the range of values for which the expansion is valid

$$(2+3x)^{-2} = \left(2\left(1+\tfrac{3x}{2}\right)\right)^{-2} = 2^{-2}\left(1+\tfrac{3x}{2}\right)^{-2}$$

$$\approx \frac{1}{4}\left[1 - 2\left(\frac{3x}{2}\right) + \frac{-\cancel{2}\cdot -3}{1\cdot\cancel{2}}\left(\frac{3x}{2}\right)^2 + \frac{-\cancel{2}\cdot-\cancel{3}\cdot-4}{1\cdot\cancel{2}\cdot\cancel{3}}\left(\frac{3x}{2}\right)^3\right]$$

$$= \frac{1}{4}\left[1 - 3x + \frac{27}{4}x^2 - \frac{27}{2}x^3\right]$$

$$= \frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 - \frac{27}{8}x^3$$

Valid for $\left|\frac{3x}{2}\right| < 1$

$$|x| < \frac{2}{3}$$

$$-\frac{2}{3} < x < \frac{2}{3}$$

Expand $\dfrac{2+x}{\sqrt{4+x}}$ as far as the term in x^3 and state the range of values for which the expansion is valid

$$\frac{2+x}{\sqrt{4\left(1+\frac{x}{4}\right)}} = \frac{2+x}{2\sqrt{1+\frac{x}{4}}} = \frac{1}{2}(2+x)\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$\approx \left(1+\frac{x}{2}\right)\left[1 + -\frac{1}{2}\left(\frac{x}{4}\right) + \frac{-\frac{1}{2}\cdot -\frac{3}{2}}{1\,.\,2}\left(\frac{x}{4}\right)^2 + \frac{-\frac{1}{2}\cdot -\frac{\cancel{3}}{2}\cdot -\frac{5}{2}}{1\,.\,2.\,\cancel{3}}\left(\frac{x}{4}\right)^3\right]$$

$$= \left(1+\frac{x}{2}\right)\left[1 - \frac{x}{8} + \frac{3}{128}x^2 - \frac{5}{1024}x^3\right]$$

$$= 1 - \frac{x}{8} + \frac{3}{128}x^2 - \frac{5}{1024}x^3$$
$$\quad + \frac{x}{2} - \frac{x^2}{16} + \frac{3}{256}x^3$$

$$= 1 + \frac{3}{8}x - \frac{5}{128}x^2 + \frac{7}{1024}x^3$$

Valid for $\left|\frac{x}{4}\right| < 1$

$|x| < 4$

$-4 < x < 4$
