Quality of Tests Exercise 8A

1) $X \sim B(10, 0.25)$ 5% sig level

$H_0: p = 0.25$
$H_1: p > 0.25$

$P(X \leq 3) = 0.7758$
$P(X \leq 4) = 0.9218 \Rightarrow P(X \geq 5) = 0.0782$
$P(X \leq 5) = 0.9802 \Rightarrow P(X \geq 6) = 0.0198$

Critical Region $\{6, 7, 8, 9, 10\}$

Prob Type I error $= 0.0198$

Given actual $p = 0.3$

$X \sim B(10, 0.3)$

Type II error $P(X \leq 5) = 0.9527$

4) $X \sim Po(6)$ 5% sig level

$H_0: \lambda = 6$
$H_1: \lambda > 6$

$P(X \leq 9) = 0.916 \Rightarrow P(X \geq 10) = 0.084$
$P(X \leq 10) = 0.9573 \Rightarrow P(X \geq 11) = 0.0427$
$< 5\%$

Critical region $X \geq 11$

$P(\text{Type I error}) = 0.0427$

$X \sim \text{Poisson}(7)$ Find $P(X \leq 10)$

$P(\text{Type II error}) = 0.9015$

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7) $X \sim Geo(0.2)$ $\quad H_0: p = 0.2$ $\quad H_1: p < 0.2$ $\quad$ 5% sig level

$P(X \le x) = 1 - (1-p)^x$

$P(X \le 13) = 1 - 0.8^{13} = 0.945$

$P(X \le 14) = 1 - 0.8^{14} = 0.956$

$\Rightarrow P(X \geq 15) = 0.044$

Critical Region $X \geq 15$

$P(\text{Type I error}) = 0.044$

Actual $p = 0.05$

$X \sim Geo(0.05)$

$P(X \le 14) = 1 - 0.95^{14}$

$P(\text{Type II error}) = 0.5123$

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11) $X \sim B(40, 0.05)$ $\quad$ 5%

$P(X \le 3) = 0.8618$

$P(X \le 4) = 0.9520 \Rightarrow P(X \geq 5) = 0.048 < 5\%$

Critical Region $X \geq 5$

$P(\text{Type I error}) = 0.0480$

$X \sim Geo(0.05)$

$P(X \leq 1) = 1 - 0.95^1 = 0.05$

Critical Region 1

$P(\text{Type I error}) = 0.05$

David $\quad X \sim Geo(.0588)$

Type II error $\quad P(X \geq 2) = 1 - 0.0588 = 0.9412$

Michael $\quad X \sim B(40, 0.0588)$

Type II error $\quad P(X \leq 4) = 0.9162$