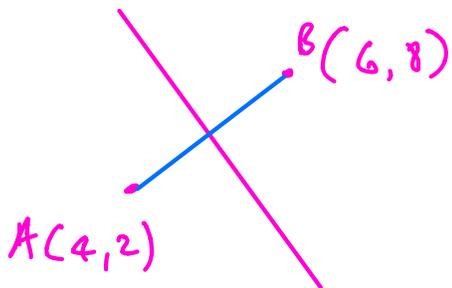


## Midpoints and Perpendicular Bisectors

Find perpendicular bisector of  $(4, 2)$  and  $(6, 8)$

$$\text{Midpoint} \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4+6}{2}, \frac{2+8}{2} \right) \\ = (5, 5)$$



$$\text{gradient } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{6-4} = 3$$

$$\text{gradient of } \perp \text{ bisector} = -\frac{1}{3} \\ \text{through } (5, 5)$$

$$y - y_1 = m(x - x_1)$$

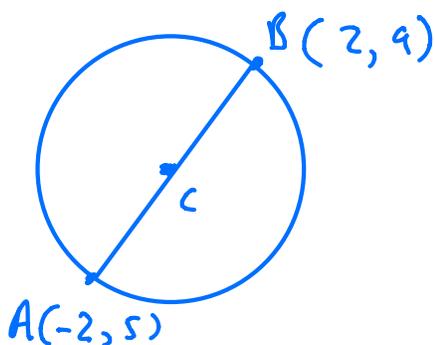
$$y - 5 = -\frac{1}{3}(x - 5)$$

$$y - 5 = -\frac{1}{3}x + \frac{5}{3}$$

$$y = -\frac{1}{3}x + \frac{20}{3}$$

---

Find the equation of circle with a diameter between  $A(-2, 5)$  and  $B(2, 9)$



$$\text{centre } C \left( \frac{-2+2}{2}, \frac{5+9}{2} \right) \\ C(0, 7)$$

$$\text{Radius} = \sqrt{(2-0)^2 + (9-7)^2} = \sqrt{8}$$

Circle eqn  $(x-0)^2 + (y-7)^2 = \sqrt{8}^2$   
 $x^2 + (y-7)^2 = 8$

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### Exercise

1) a) Centre (3, 2) radius 4

$$(x-3)^2 + (y-2)^2 = 4^2$$

---

1 e) Centre  $(-2\sqrt{2}, -3\sqrt{2})$  radius 1

$$(x+2\sqrt{2})^2 + (y+3\sqrt{2})^2 = 1$$

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2 a)  $(x+5)^2 + (y-4)^2 = 9^2$

Centre  $(-5, 4)$  radius 9

---

2 e)  $(x-3\sqrt{5})^2 + (y+\sqrt{5})^2 = 27$

Centre  $(3\sqrt{5}, -\sqrt{5})$  radius =  $3\sqrt{3}$

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3 a)  $(x-2)^2 + (y-5)^2 = 13$

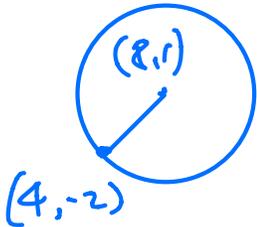
Point (4, 8)  $(4-2)^2 + (8-5)^2$

$$= 2^2 + 3^2$$

$$= 4 + 9 = 13 \checkmark$$

---

4)  $(4, -2)$  on circle centre  $(8, 1)$



$$\text{radius} = \sqrt{(8-4)^2 + (1-(-2))^2}$$
$$\sqrt{4^2 + 3^2} = 5$$

$$\text{Eqn } (x-8)^2 + (y-1)^2 = 5^2$$

---

10) a)  $x^2 + y^2 - 2x + 8y - 8 = 0$

$$(x-1)^2 - 1 + (y+4)^2 - 16 - 8 = 0$$

$$(x-1)^2 + (y+4)^2 = 25$$

Centre  $(1, -4)$  radius = 5

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Exercise 6c 1b, 2b, 3b, 5, 7, 9, 10b, c, d, e

11, 13, Finish for Homework

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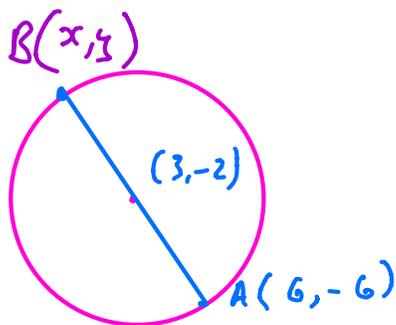
11 A circle has equation  $(x - 3)^2 + (y + 2)^2 = 25$ .

- (i) State the coordinates of the centre of this circle and its radius. [2]
- (ii) Verify that the point A with coordinates  $(6, -6)$  lies on this circle. Show also that the point B on the circle for which AB is a diameter has coordinates  $(0, 2)$ . [3]
- (iii) Find the equation of the tangent to the circle at A. [4]
- (iv) A second circle touches the original circle at A. Its radius is 10 and its centre is at C, where BAC is a straight line. Find the coordinates of C and hence write down the equation of this second circle. [3]

i) Centre  $(3, -2)$  radius 5

ii) 
$$(6-3)^2 + (-6+2)^2$$
$$= 3^2 + 4^2$$
$$= 25 \checkmark \therefore \text{on circle}$$

---



$B(0, 2)$

$(3, 2)$  is midpoint of AB

$$\left( \frac{6+x}{2}, \frac{-6+y}{2} \right) = (3, 2)$$

$$\frac{6+x}{2} = 3$$

$$6+x = 6$$
$$x = 0$$

$$\frac{-6+y}{2} = 2$$

$$-6+y = 4$$
$$y = 2$$

iii) Centre  $(3, 2)$        $A(6, -6)$

radius to A gradient  $\frac{-6-2}{6-3} = -\frac{8}{3}$

$$\text{Gradient at A} = +\frac{3}{8}$$

$$y - y_1 = m(x - x_1)$$

$$y - -6 = \frac{3}{8}(x - 6)$$

$$y + 6 = \frac{3}{8}x - \frac{18}{8}$$

$$y = \frac{3}{8}x - \frac{42}{8}$$

$$y = \frac{3}{8}x - \frac{21}{4}$$

