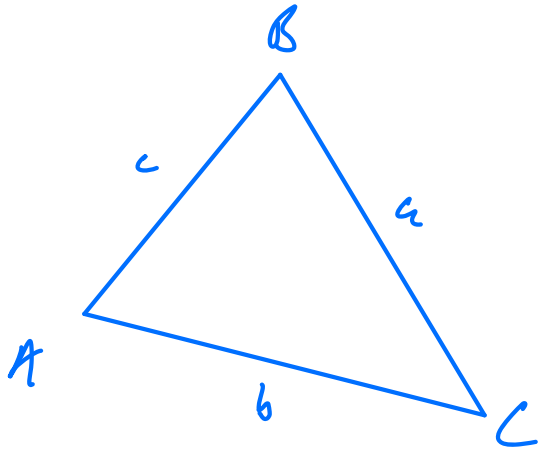
Cosine Rule



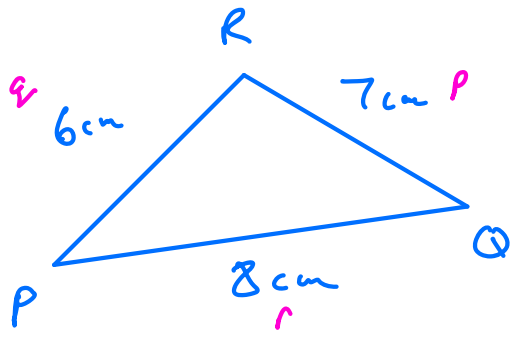
$a^2 = b^2 + c^2 - 2bc\cos A$

to find a side

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

to find an angle

Finding Angles



Find all 3 angles

$\cos P = \dfrac{q^2 + r^2 - p^2}{2qr}$

$\cos P = \dfrac{(6^2 + 8^2 - 7^2)}{(2 \times 6 \times 8)}$

$\cos P = \dfrac{17}{32}$

$P = \cos^{-1}\left(\dfrac{17}{32}\right) = 57.9^\circ$

$\cos Q = \dfrac{p^2 + r^2 - q^2}{2pr}$

$\cos Q = \dfrac{(7^2 + 8^2 - 6^2)}{(2 \times 7 \times 8)}$

$$\cos Q = \frac{11}{16}$$

$$Q = \cos^{-1}\left(\frac{11}{16}\right) = 46.6^\circ$$

$$\cos R = \frac{p^2+q^2-r^2}{2pq} = \frac{(7^2+6^2-8^2)}{(2\times7\times6)}$$

$$\cos R = \frac{1}{4}$$

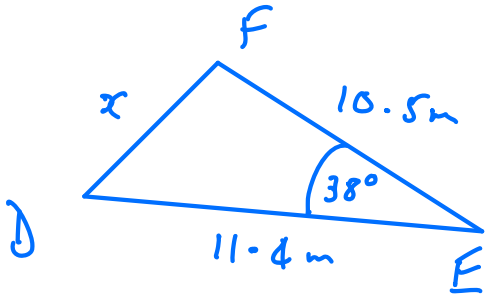
$$R = \cos^{-1}\left(\frac{1}{4}\right) = 75.5^\circ$$

Check $P + Q + R$

$$= \begin{array}{r} 57.9 \\ 46.6 \\ 75.5 \\ \hline 180.0 \end{array} +$$

✓

Finding a side



Find x

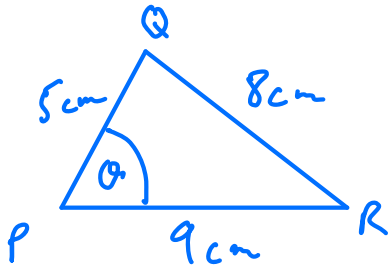
$$x^2 = 11.4^2 + 10.5^2 - 2\times11.4\times10.5\cos38^\circ$$

$$x^2 = 51.56$$

$$x = \sqrt{51.56}$$

$$x = 7.18\text{ m}$$

Exercise on Cosine Rule

1) 

Find θ

$$\cos\theta = \frac{(9^2+5^2-8^2)}{(2\times 9\times 5)}$$

$$\cos\theta = \frac{7}{15}$$

$$\theta = \cos^{-1}\left(\frac{7}{15}\right) = 62.2^\circ$$

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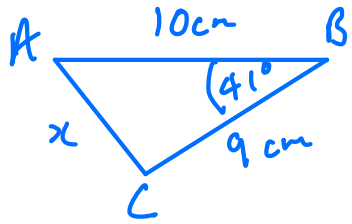
2) 

Find x

$$x^2 = 9^2+10^2-2\times 9\times 10\cos 41^\circ$$

$$x^2 = 41.15$$

$$x = \sqrt{41.15} = 6.72\text{ cm}$$

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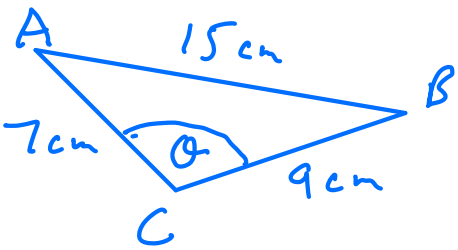
3) 

Find θ

$$\cos\theta = \frac{(7^2+9^2-15^2)}{(2\times 7\times 9)}$$

$$\cos\theta = -\frac{95}{126}$$

$$\theta = \cos^{-1}\left(-\frac{95}{126}\right) = 138.9^\circ$$

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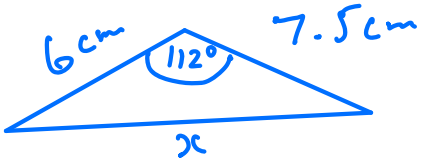
4) 

Find x

$$x^2 = 6^2+7.5^2-2\times 6\times 7.5\cos 112^\circ$$

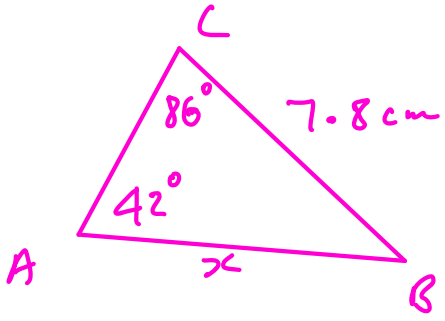
$$x^2 = 125.96$$

$$x = \sqrt{125.96}$$

$$x = 11.2\text{ cm}$$

Sine Role

Further Examples

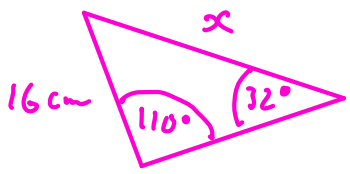
1) 

Sine Rule

$$\frac{x}{\sin 86^\circ} = \frac{7.8}{\sin 42^\circ}$$

$$x = \frac{7.8}{\sin 42}\times \sin 86 = 11.6\text{ cm}$$

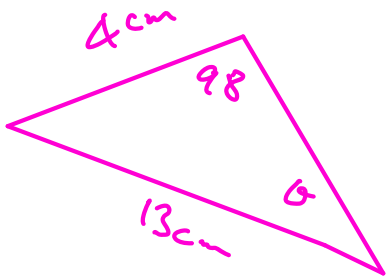
2)



$$\frac{x}{\sin 110} = \frac{16}{\sin 32}$$

$$x = \frac{16}{\sin 32} \times \sin 110 = 28.4 \text{ cm}$$

3)



$$\frac{4}{\sin\theta} = \frac{13}{\sin 98}$$

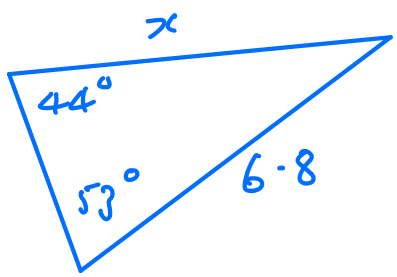
$$\frac{\sin\theta}{4} = \frac{\sin 98}{13}$$

$$\sin\theta = \frac{\sin 98}{13} \times 4 = 0.3047$$

$$\theta = \sin^{-1}(0.3047) = 17.7°$$

Exercise

1)



$$\frac{x}{\sin 53°} = \frac{6.8}{\sin 44°}$$

$$x = \frac{6.8}{\sin 44°} \times \sin 53° = 7.82 \text{ cm}$$

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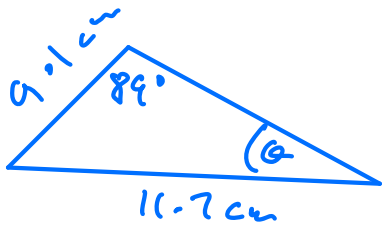
2)



$$\frac{9.1}{\sin\theta} = \frac{11.7}{\sin 89°}$$

$$\frac{\sin\theta}{9.1} = \frac{\sin 89}{11.7}$$

$$\sin\theta = \frac{\sin 89}{11.7} \times 9.1 = 0.7777$$

$$\theta = \sin^{-1}(0.7777) = 51.0°$$
