2. A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(Total 8 marks)

$$v = 3t^2 - 4t + 3$$

$$s = \int v\,dt = t^3 - 2t^2 + 3t + c$$

$t = 0$
$s = 0$

$$0 = 0^3 - 2 \times 0^2 + 3 \times 0 + c \Rightarrow c = 0$$

$$\underline{s = t^3 - 2t^2 + 3t}$$

Min velocity when $\frac{dv}{dt} = 0$

$$\frac{dv}{dt} = 6t - 4$$

$$6t - 4 = 0$$
$$6t = 4$$
$$t = \frac{4}{6}$$
$$t = \frac{2}{3}$$

$$t = \frac{2}{3} \Rightarrow s = \left(\frac{2}{3}\right)^3 - 2\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right)$$

$$s = \frac{38}{27}\text{ m} = 1.41\text{ m}$$

3. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2.$$

(a) Find the maximum value of v.

(4)

(b) Find the time taken for P to return to O.

(5)

(Total 9 marks)

a) $V = 8t - t^2$

$$\frac{dV}{dt} = 8 - 2t$$

Max when $\frac{dv}{dt} = 0 \Rightarrow 8 - 2t = 0$

$8 = 2t$

$t = 4$ s

Check max $\frac{d^2v}{dt^2} = -2 < 0 \therefore$ max

when $t = 4$ $V = 8(4) - 4^2 = 16$ ms^{-1}

$\underline{\text{Max } v = 16 \text{ ms}^{-1}}$

b) $s = \int v\,dt = \int (8t - t^2)\,dt$

$$= 4t^2 - \frac{t^3}{3} + c$$

$\left.\begin{matrix} t = 0 \\ s = 0 \end{matrix}\right\} \Rightarrow c = 0$

$$\underline{s = 4t^2 - \frac{t^3}{3}}$$

Back at O when $s = 0$

$$s = t^2\left(4 - \frac{t}{3}\right)$$

$s = 0 \quad \Rightarrow t = 0 \text{ or } 4 - \frac{t}{3} = 0$

$4 = \frac{t}{3}$

$12 = t$

Back at O when $t = 12$ s

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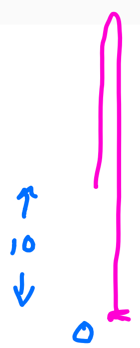
2. At time $t = 0$, a particle is projected vertically upwards with speed u m s^{-1} from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed 17.5 m s^{-1}. Find

(a) the value of u,

(3)

(b) the value of T.

(4)



$v^2 = u^2 + 2a(s - s_0)$

$17.5^2 = u^2 - 19.6(0 - 10)$

$17.5^2 - 196 = u^2$

$\frac{441}{4} = u^2$

$u = 10.5$ ms^{-1}

b) $v = u + at$

$-17.5 = 10.5 - 9.8T$

$9.8T = 10.5 + 17.5$

$9.8T = 28$

$T = \frac{28}{9.8} = \frac{20}{7}$

$T = 2\frac{6}{7}$ s

$s - s_0 = ut + \frac{1}{2}at^2$

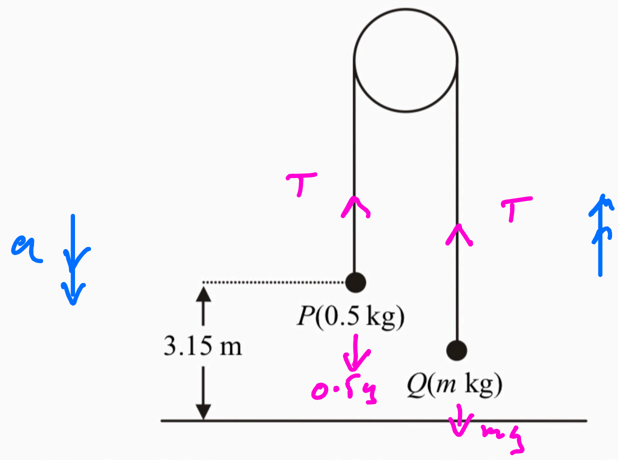
$0 - 10 = 10.5T - 4.9T^2$

$4.9T^2 - 10.5T - 10 = 0$

By calc $T = \frac{20}{7}$ or $\cancel{T = -\frac{5}{7}}$

$T = 2\frac{6}{7}$ s

6.



Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in the diagram above. After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.

(a) Show that the acceleration of P as it descends is 2.8 m s^{-2}.

(3)

(b) Find the tension in the string as P descends.

(3)

a) P travels 3.15m in 1.5 s

$$s = ut + \frac{1}{2}at^2$$

$$3.15 = 0 + \frac{1}{2}a \times 1.5^2$$

$$2 \times \frac{3.15}{1.5^2} = a \qquad a = 2.8 \text{ ms}^{-2}$$

b) N2L for P

$$0.5g - T = 0.5 \times 2.8$$

$$0.5 \times 9.8 - 0.5 \times 2.8 = T$$

$$\frac{7}{2} = T$$

$$T = 3.5\text{ N}$$

(c) Show that $m = \frac{5}{18}$.

(4)

(d) State how you have used the information that the string is inextensible.

(1)

c) N2L for Q

$$T - mg = ma$$

$$T = m(a+g)$$

$$\frac{T}{a+g} = m$$

$$m = \frac{3.5}{2.8+9.8} = \frac{3.5}{12.6} = \frac{5}{18} \text{ kg}$$

d) acceleration magnitude same for P and Q