Solutions

(Total 6 marks)

Q1.

The curve C has the equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$

(a) Use algebra to show that C and L do not intersect.

(4)

(b) In figure 1 sketch C and L on the same diagram, showing the coordinates of the points at which C and L

meet the axes.

(4)

(Total 8 marks)

$y = x(5 - x)$ ①

$2y = 5x + 4$ ②

① × 2 $2y = 2x(5 - x)$ ③

At point of intersection

$5x + 4 = 2x(5 - x)$

$5x + 4 = 10x - 2x^2$

$2x^2 - 10x + 5x + 4 = 0$

$2x^2 - 5x + 4 = 0$

Discriminant $b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4$

$= 25 - 32$

$= -7 < 0$

$\therefore$ no roots so point of intersection

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

(b) Calculate the discriminant of $4x - 5 - x^2$

a) $4x - 5 - x^2$

$= -\left[x^2 - 4x + 5\right]$

$= -\left[(x-2)^2 + 5 - 4\right]$

$= -\left[(x-2)^2 + 1\right]$

$= -1 - (x-2)^2$

$p = -2$

$q = -1$

b) Discriminant of $-x^2 + 4x - 5$

$b^2 - 4ac = 4^2 - 4(-1)(-5)$

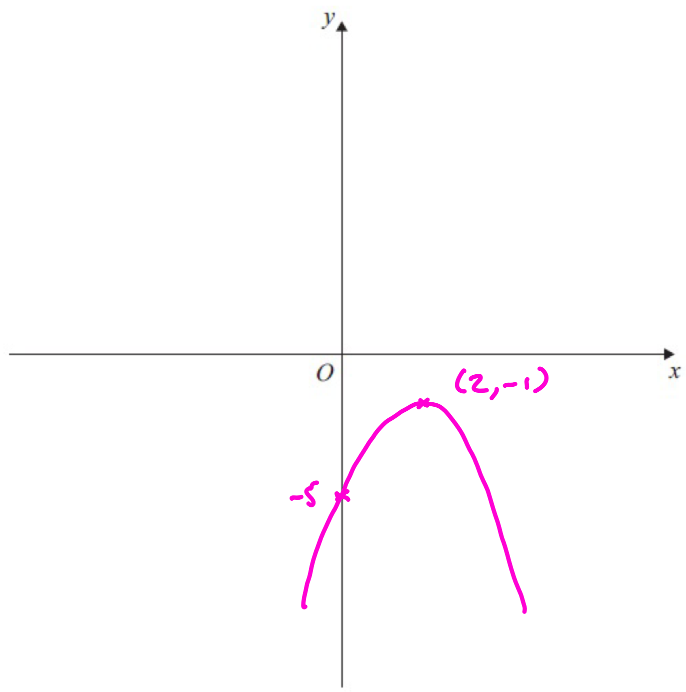
$= 16 - 20$

$= -4$

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

when $x = 0$
$y = -5$



(Total 8 marks)

Q5.

The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

(Total 7 marks)

for distinct real solutions $b^2 - 4ac > 0$

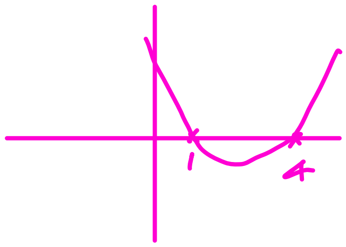
$$\Rightarrow \quad 4^2 - 4k(5-k) > 0$$

$$16 - 20k + 4k^2 > 0$$

$\div 4$ $$\underline{k^2 - 5k + 4 > 0}$$

b) $$(k-1)(k-4) > 0$$

$y = k^2 - 5k + 4$



Either $k > 4$

or $k < 1$

Q7.

$$f(x) = x^2 + 4kx + (3+11k), \text{ where } k \text{ is a constant.}$$

(a) Express f(x) in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation f(x) = 0 has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of y = f(x), showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)

(Total 10 marks)

a) $f(x) = x^2 + 4kx + (3 + 11k)$

$= (x + 2k)^2 + 3 + 11k - 4k^2$

$= (x + 2k)^2 + (3 + 11k - 4k^2)$

$p = 2k$

$q = 3 + 11k - 4k^2$

b) No real roots $\Rightarrow b^2 - 4ac < 0$

$$\Rightarrow (4k)^2 - 4 \times 1 \times (3 + 11k) < 0$$

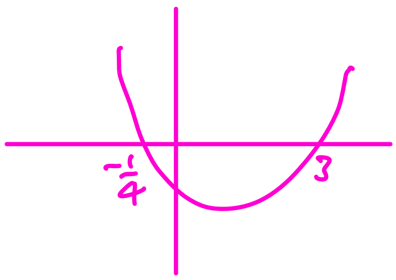
$$16k^2 - 12 - 44k < 0$$

$$4k^2 - 3 - 11k < 0$$

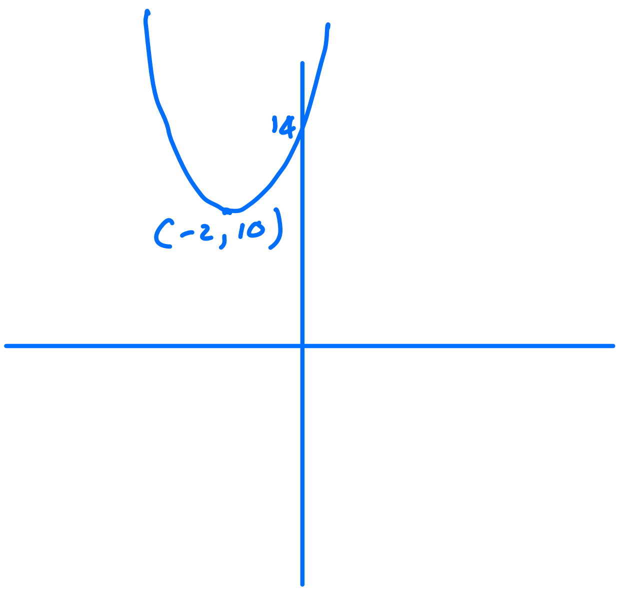
$$4k^2 - 11k - 3 < 0$$

$$(4k+1)(k-3) < 0$$

$y = 4k^2 - 11k - 3$



$$-\frac{1}{4} < k < 3$$



$y = x^2 + 4x + 14$

$y = (x+2)^2 + 10$

INDEPENDENT CLASS WORK

Q2.

The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2 + 2k - 3 > 0$$

(3)

(b) Find the set of possible values of k.

(4)

(Total 7 marks)

a)

For two real roots $b^2 - 4ac > 0$

$$\Rightarrow \quad (k-3)^2 - 4 \times 1 \times (3-2k) > 0$$

$$k^2 - 6k + 9 - 12 + 8k > 0$$

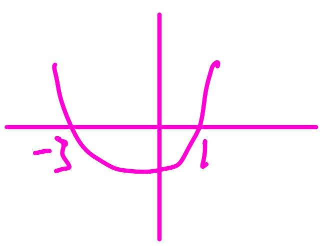
$$\underline{k^2 + 2k - 3 > 0}$$

b)

$$k^2 + 2k - 3 > 0$$

$$(k+3)(k-1) > 0$$

$y = k^2 + 2k - 3$



Either $k > 1$

or $k < -3$

Q4.

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

(Total 6 marks)

a) Discriminant $= b^2 - 4ac$

$$= (k+3)^2 - 4 \times 1 \times k$$

$$= k^2 + 6k + 9 - 4k$$

$$= \underline{k^2 + 2k + 9}$$

b)

$$k^2 + 2k + 9$$

$$= (k+1)^2 + 9 - 1$$

$$= \underline{(k+1)^2 + 8}$$

c) $(k+1)^2 \geq 0$ for all values of k

$\Rightarrow (k+1)^2 + 8 \geq 8$ for all values of k

$f(x) = 0$ has real roots for all k since discriminant is always positive

Q6.

(a) Show that $x^2 + 6x + 11$ can be written as

$$(x+p)^2 + q$$

where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

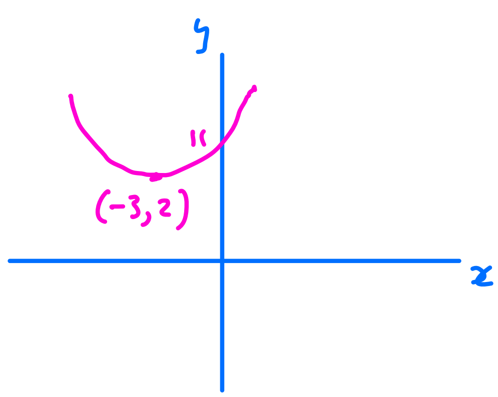
(Total 6 marks)

a) $x^2 + 6x + 11$

$= (x+3)^2 + 11 - 9$

$= (x+3)^2 + 2$

b)



c) Discriminant $= b^2 - 4ac$ $= 6^2 - 4 \times 1 \times 11$

$= 36 - 44$

$= -8$

Q8.

The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

(Total 4 marks)

Equal roots $\Rightarrow$ $b^2 - 4ac = 0$

$$\Rightarrow \quad (3p)^2 - 4 \times 1 \times p = 0$$

$$9p^2 - 4p = 0$$

$$p(9p - 4) = 0$$

Given that $p \neq 0$

$$\therefore \quad 9p - 4 = 0$$

$$9p = 4$$

$$\underline{p = \frac{4}{9}}$$

Q10.

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

a) $2(3x + 4) > 1 - x$

$$6x + 8 > 1 - x$$

$$6x + x > 1 - 8$$

$$7x > -7$$

$$x > -\frac{7}{7}$$

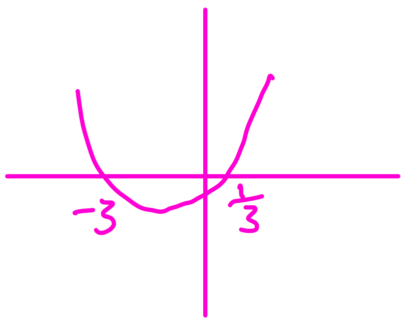
$$x > -1$$

b)

$$3x^2 + 8x - 3 < 0$$

$$(3x - 1)(x + 3) < 0$$

$$y = 3x^2 + 8x - 3$$



$$-3 < x < \frac{1}{3}$$

Q9.

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total 7 marks)

a) $(k+3)x^2 + 6x + (k-5) = 0$

For two distinct real roots

$$b^2 - 4ac > 0$$

$$\Rightarrow \quad 6^2 - 4(k+3)(k-5) > 0$$

$$36 - 4(k^2 + 3k - 5k - 15) > 0$$

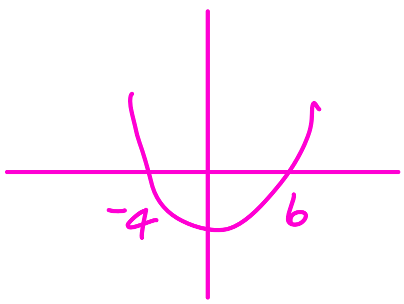
$$36 - 4k^2 - 12k + 20k + 60 > 0$$

$$4k^2 + 12k - 20k - 96 < 0$$

$$4k^2 - 8k - 96 < 0$$

$$\underline{k^2 - 2k - 24 < 0}$$

b) $y = k^2 - 2k - 24$ $\qquad (k+4)(k-6) < 0$



$$\underline{-4 < k < 6}$$