# Quadratic Exam Questions

## Questions

Q1.

The curve $C$ has the equation $y = x(5 - x)$ and the line $L$ has equation $2y = 5x + 4$

(a) Use algebra to show that $C$ and $L$ do not intersect.

**(4)**

(b) In figure 1 sketch $C$ and $L$ on the same diagram, showing the coordinates of the points at which $C$ and $L$

meet the axes.

**(4)**

**(Total 8 marks)**

Q2.

The equation $x^2 + (k - 3)x + (3 - 2k) = 0$, where $k$ is a constant, has two distinct real roots.

(a) Show that $k$ satisfies

$$k^2 + 2k - 3 > 0$$

**(3)**

(b) Find the set of possible values of $k$.

**(4)**

**(Total 7 marks)**

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where $p$ and $q$ are integers.

(a) Find the value of $p$ and the value of $q$.

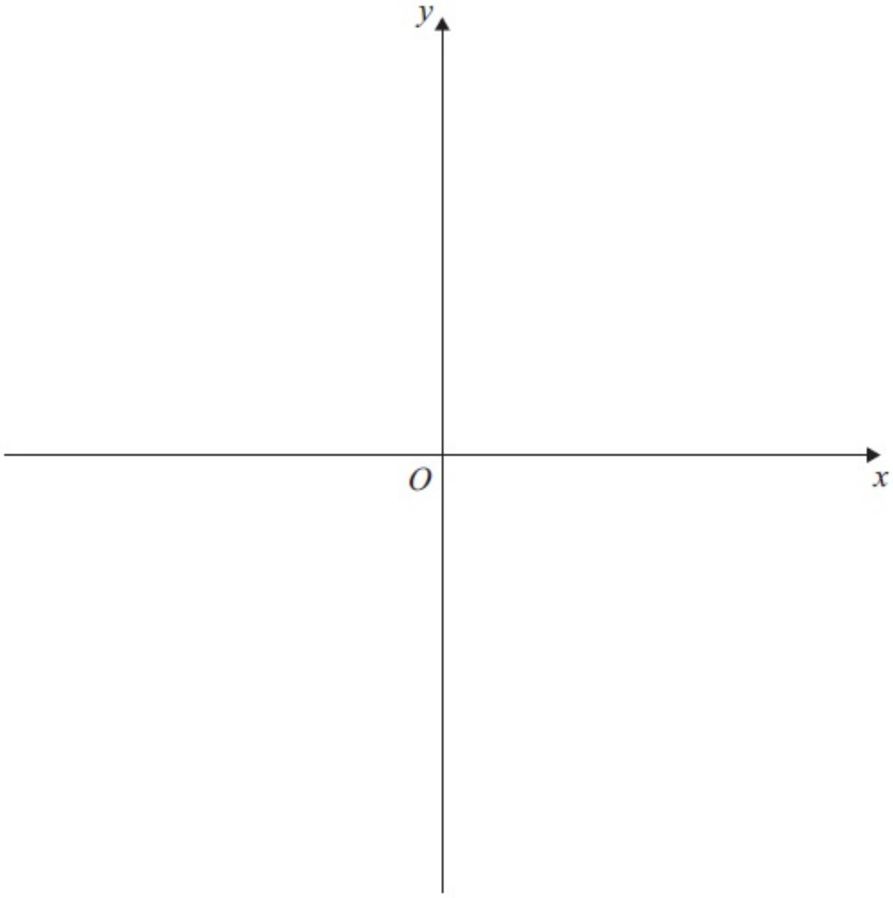
**(3)**

(b) Calculate the discriminant of $4x - 5 - x^2$

**(2)**

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

**(3)**




**(Total 8 marks)**

Q4.

$$f(x) = x^2 + (k+3)x + k$$

where $k$ is a real constant.

(a) Find the discriminant of $f(x)$ in terms of $k$.

**(2)**

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where $a$ and $b$ are integers to be found.

**(2)**

(c) Show that, for all values of $k$, the equation $f(x) = 0$ has real roots.

**(2)**

**(Total 6 marks)**

Q5.

The equation $kx^2 + 4x + (5 - k) = 0$, where $k$ is a constant, has 2 different real solutions for $x$.

(a) Show that $k$ satisfies

$$k^2 - 5k + 4 > 0.$$

**(3)**

(b) Hence find the set of possible values of $k$.

**(4)**

**(Total 7 marks)**

Q6.

(a) Show that $x^2 + 6x + 11$ can be written as

$$(x+p)^2 + q$$

where $p$ and $q$ are integers to be found.

**(2)**

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

**(2)**

(c) Find the value of the discriminant of $x^2 + 6x + 11$

**(2)**

**(Total 6 marks)**

Q7.

$$f(x) = x^2 + 4kx + (3+11k), \text{ where } k \text{ is a constant.}$$

(a) Express $f(x)$ in the form $(x + p)^2 + q$, where $p$ and $q$ are constants to be found in terms of $k$.

**(3)**

Given that the equation $f(x) = 0$ has no real roots,

(b) find the set of possible values of $k$.

**(4)**

Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

**(3)**

**(Total 10 marks)**

Q8.

The equation $x^2 + 3px + p = 0$, where $p$ is a non-zero constant, has equal roots.

Find the value of $p$.

**(4)**

**(Total 4 marks)**

Q9.

The equation

$$(k + 3)\, x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for $x$.

(a) Show that $k$ satisfies

$$k^2 - 2k - 24 < 0$$

**(4)**

(b) Hence find the set of possible values of $k$.

**(3)**

**(Total 7 marks)**

Q10.

Find the set of values of $x$ for which

(a) $2(3x + 4) > 1 - x$

**(2)**

(b) $3x^2 + 8x - 3 < 0$

**(4)**

# Solutions

**(Total 6 marks)**

Q1.

The curve $C$ has the equation $y = x(5 - x)$ and the line $L$ has equation $2y = 5x + 4$

(a) Use algebra to show that $C$ and $L$ do not intersect.

**(4)**

(b) In figure 1 sketch $C$ and $L$ on the same diagram, showing the coordinates of the points at which $C$ and $L$

meet the axes.

**(4)**

**(Total 8 marks)**

$y = x(5 - x)$ ①

$2y = 5x + 4$ ②

① $\times 2$ $\quad 2y = 2x(5 - x)$ ③

At point of intersection

$$5x + 4 = 2x(5 - x)$$

$$5x + 4 = 10x - 2x^2$$

$$2x^2 - 10x + 5x + 4 = 0$$

$$2x^2 - 5x + 4 = 0$$

Discriminant $b^2 - 4ac = (-5)^2 - 4 \times 2 \times 4$

$= 25 - 32$

$= -7 < 0$

$\therefore$ no roots so point of intersection

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where $p$ and $q$ are integers.

(a) Find the value of $p$ and the value of $q$.

(b) Calculate the discriminant of $4x - 5 - x^2$

a) $4x - 5 - x^2$

$= -\left[x^2 - 4x + 5\right]$

$= -\left[(x-2)^2 + 5 - 4\right]$

$= -\left[(x-2)^2 + 1\right]$

$= -1 - (x-2)^2$

$p = -2$

$q = -1$

b) Discriminant of $-x^2 + 4x - 5$

$b^2 - 4ac = 4^2 - 4(-1)(-5)$

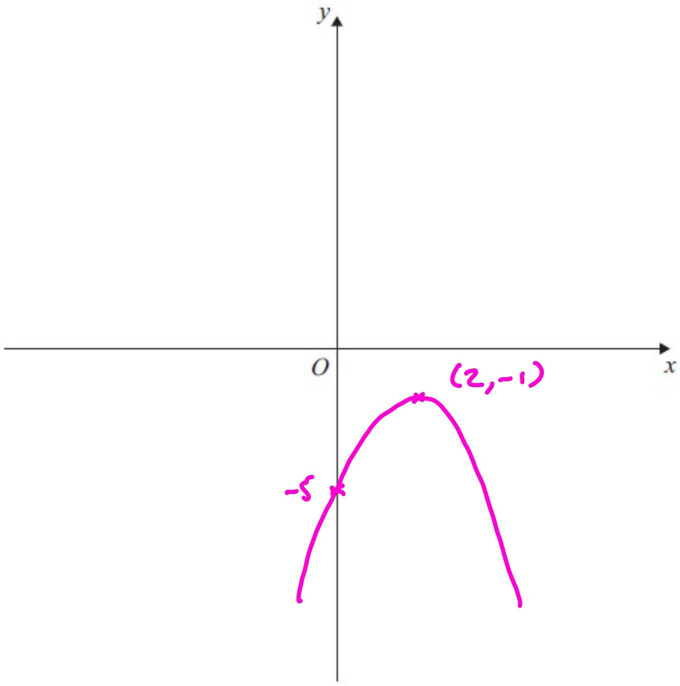
$= 16 - 20$

$= -4$

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

**(3)**

when $x = 0$
$y = -5$




**(Total 8 marks)**

Q5.

The equation $kx^2 + 4x + (5 - k) = 0$, where $k$ is a constant, has 2 different real solutions for $x$.

(a) Show that $k$ satisfies

$$k^2 - 5k + 4 > 0.$$

**(3)**

(b) Hence find the set of possible values of $k$.

**(4)**

**(Total 7 marks)**

for distinct real solutions $b^2 - 4ac > 0$

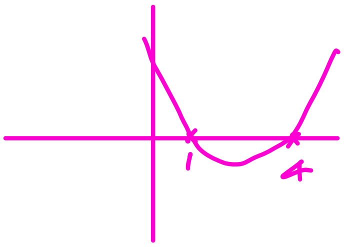
$$\Rightarrow \quad 4^2 - 4k(5-k) > 0$$

$$16 - 20k + 4k^2 > 0$$

$\div 4$ $$\underline{k^2 - 5k + 4 > 0}$$

b) $$(k-1)(k-4) > 0$$

$y = k^2 - 5k + 4$




Either $k > 4$

or $k < 1$

Q7.

$$f(x) = x^2 + 4kx + (3+11k), \text{ where } k \text{ is a constant.}$$

(a) Express f($x$) in the form $(x + p)^2 + q$, where $p$ and $q$ are constants to be found in terms of $k$.

**(3)**

Given that the equation f($x$) = 0 has no real roots,

(b) find the set of possible values of $k$.

**(4)**

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Given that $k$ = 1,

(c) sketch the graph of $y$ = f($x$), showing the coordinates of any point at which the graph crosses a coordinate axis.

**(3)**

**(Total 10 marks)**

a)
$$\begin{aligned} f(x) &= x^2 + 4kx + (3+11k) \\ &= (x+2k)^2 + 3 + 11k - 4k^2 \\ &= (x+2k)^2 + (3 + 11k - 4k^2) \end{aligned}$$

$p = 2k$

$q = 3 + 11k - 4k^2$

b) No real roots $\Rightarrow b^2 - 4ac < 0$

$$\Rightarrow (4k)^2 - 4 \times 1 \times (3 + 11k) < 0$$

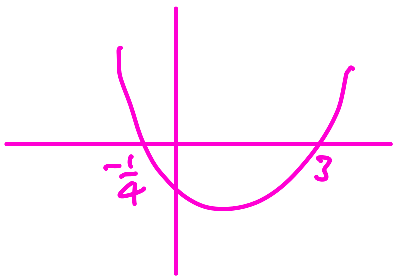
$$16k^2 - 12 - 44k < 0$$

$$4k^2 - 3 - 11k < 0$$

$$4k^2 - 11k - 3 < 0$$

$$(4k+1)(k-3) < 0$$

$y = 4k^2 - 11k - 3$




$$-\frac{1}{4} < k < 3$$

$y = x^2 + 4x + 14$

$y = (x+2)^2 + 10$