

## Growth and Decay Problems 2

### Exercise 22.5A

- 1 For each account in the table below, find the compound interest earned.

Acc	Original amount	Compound interest rate	Number of years
a	£250	4% per year	6
b	£840	2.5% per 6 months	5
c	£4500	1.25% per 3 months	3

- 2 A building society offers two accounts: Karen says that they would give the same interest on an investment. Is Karen correct? Explain your answer.

#### Easy Saver

4% interest added at the end of each year

#### Half-yearly saver

2% interest added at the end of every 6 months

- 3 A road planner uses the formula  $2400 \times 1.08^n$  to estimate the number of vehicles per day that will travel on a new road  $n$  months after it opens.
- Describe two assumptions the planner has made.
  - Sketch a graph to show what the planner expects to happen.
  - Give reasons why the planner's assumptions may not be appropriate.
- 4 There are 250 rare trees in a forest, but each year the number of trees falls by 30%. A woodland trust aims to plant 60 more trees in the forest at the end of each year.
- Show that  $T_{n+1} = 0.7T_n + 60$  where  $T_n$  denotes the number of trees in the forest after  $n$  years.
  - Work out the number of trees after 5 years.
  - Sketch a graph to show how the number of trees varies in this time. State any assumptions you make.
- 5 Ben takes out a loan for £500. Interest of 2% is added to the amount owing at the end of each month, then Ben pays off £90 or all the amount owing when it is less than £90.
- How long will it take Ben to pay off the loan? Show your working.

- Work out the percentage interest that Ben will pay on the loan of £500.
- 6 Sally invests £8000 in an account that pays 3.5% interest at the end of each year. Sally has to pay 20% tax on this interest. Calculate how much Sally will have in her account at the end of 4 years.
- 7 Liam finds a formula for the compound interest earned by £ $P$  invested for 6 years at a rate of 4.5%. Here is Liam's method.

$$\text{Interest in 1 year} = 0.045 \times \text{£}P$$

$$\text{Interest for 6 years} = 6 \times 0.045 \times \text{£}P = \text{£}0.27P$$

- Why is Liam's method incorrect?
  - Find a correct formula.
  - After 6 years the interest earned is £1934.46. Find, to the nearest one pound, the original amount £ $P$ .
- 8 Find the minimum rate of interest for an investment of £500 to grow to £600 in 6 years.
- 9 Tanya measures the temperature of a cup of coffee as it cools.

Time $t$ (min)	0	10	20	30	40	50	60
Temperature $T$ (°C)	85	68	55	45	39	34	31

- Use Tanya's data to draw a graph.
  - Find the rate at which the coffee is cooling after half an hour.
- 10 Tanya says  $T = 20 + 65 \times 0.97^t$  is a good model of the data.
- Is Tanya correct? Show how you decide.
  - Explain each term in Tanya's model.
- \*10 The half-life of caesium-137 is 30 years.
- Show that when 1 kilogram of caesium-137 decays, the amount left after  $t$  years is  $f(t) = 2^{-\frac{t}{30}}$  kg
  - Sketch a graph of amount against time.
  - Describe how the function and graph would change if  $f(t)$  was given in terms of grams instead of kilograms.



Classwork Q5 →

5 Ben takes out a loan for £500. Interest of 2% is added to the amount owing at the end of each month, then Ben pays off £90 or all the amount owing when it is less than £90.

a How long will it take Ben to pay off the loan? Show your working.

$$\begin{array}{llll} \text{end month 1} & 500 \times 1.02 - 90 & = & 420 \\ 2 & 420 \times 1.02 - 90 & = & 338.4 \\ 3 & 338.4 \times 1.02 - 90 & = & 255.17 \\ 4 & 255.168 \times 1.02 - 90 & = & 170.27 \\ 5 & 170.27136 \times 1.02 - 90 & = & 83.68 \\ 6 & 83.68 \times 1.02 & = & \pounds 85.35 \\ & \text{which he pays off} & & \end{array}$$

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5 b Work out the percentage interest that Ben will pay on the loan of £500.

$$\begin{aligned} \text{Total payments} &= 5 \times \pounds 90 + \pounds 85.35 \\ &= \pounds 535.35 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= \pounds 35.35 & \% \text{ interest} &= \frac{35.35}{500} \times 100 \\ & & &= 7.07\% \end{aligned}$$

- 6 Sally invests £8000 in an account that pays 3.5% interest at the end of each year. Sally has to pay 20% tax on this interest. Calculate how much Sally will have in her account at the end of 4 years.

$$3.5 \times 0.8 = 2.8$$

so effective rate of interest = 2.8%

$$8000 \times 1.028^4 = £8934.34$$

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