Growth and Decay

Exercise 22.5S

1 Find the decimal multiplier for each percetange change.

a Increase of 25% b Decrease of 25%

c Increase of 2.5% d Decrease of 2.5%

2 The number of trout in a lake is 800. The number decreases by 15% each year.

a Draw a graph to illustrate the fall in the population over the next 8 years.

b i Find a formula for the number of trout after n years.

ii Use your formula to check three values on your graph.

3 A bacteria population doubles every 20 minutes.

a Starting with 1 bacterium, draw a graph of the population growth over 3 hours.

b Estimate the number of bacteria after

i 150 minutes ii 170 minutes

c What has happened to the population between 150 minutes and 170 minutes?

4 The table gives information about the population growth of two bacteria colonies.

Colony	Population now	Increase per hour
A	200	50%
B	400	35%

a Show that the population of Colony A after n hours is 200×1.5^n

b Find an expression for the population of Colony B after n hours.

c When does the population of Colony A become bigger than that of Colony B?

5 The value of a new car is £16 000. The car loses 15% of its value at the start of each year.

a Find a formula for the value of the car after n years.

b Find the value of the car after 4 years.

c After how many complete years will the car's value drop below £4000?

6 The population of a town is 52 000. The population increases by 1.5% each year.

a Find the population after 6 years.

b When will the population reach 60 000?

7 Sadie invests £2000 in a savings account. The bank adds 4% compound interest at the end of each year. Sadie does not add or take any money from the account for 10 years.

a Copy and extend this table to show how Sadie's investment grows.

End of year	Amount in the account (£)
1	$2000 \times 1.04 = 2080$
2	

b Work out the percentage interest that Sadie's investment earns in 10 years.

c £P is invested with compound interest r% added at the end of each time period.

i Show that the total amount at the end of n time periods is $A = P\left(1 + \frac{r}{100}\right)^n$

A time period is usually a year or a number of months.

ii Use this formula to check the last amount in your table in part **a**.

8 The half-life of a radioactive substance is the time it takes for the amount to go down to half of the original amount.

After how many half-lives will there be less than 1% of the radioactive substance left?

*9 The table shows how the population of the world has grown since 1900.

a Draw a graph of this data.

b A growth function $P = 1.65 \times 1.0125^{(y-1900)}$ where P is the population in billions in year y has been suggested as a model.

i Show this function on your graph.

ii What annual percentage increase in world population does the model assume?

Year	Population (billions)
1900	1.65
1910	1.75
1920	1.86
1930	2.07
1940	2.30
1950	2.56
1960	3.04
1970	3.71
1980	4.45
1990	5.29
2000	6.09
2010	6.87

1070, 1238 SEARCH

Continue with Q 1, 3, 4, 5, 6, 8

Omit any graphs

1 Find the decimal multiplier for each percetange change.

a Increase of 25% **b** Decrease of 25%

c Increase of 2.5% **d** Decrease of 2.5%

a) 1.25 b) 0.75

c) 1.025 d) 0.975

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3 A bacteria population doubles every 20 minutes.

a Starting with 1 bacterium, draw a graph of the population growth over 3 hours.

b Estimate the number of bacteria after

i 150 minutes **ii** 170 minutes

c What has happened to the population between 150 minutes and 170 minutes?

a)

Time	Bacteria
0	1
20 min	2
40 min	4
60 min	8
80 min	16
100 min	32
120 min	64
140 min	128
160 min	256
180 min	512

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4 The table gives information about the population growth of two bacteria colonies.

Colony	Population now	Increase per hour
A	200	50%
B	400	35%

a Show that the population of Colony A after n hours is 200×1.5^n

b Find an expression for the population of Colony B after n hours.

c When does the population of Colony A become bigger than that of Colony B?

a) increase of 50% requires multiplier of 1.5 applied once every hour for n hours

so 200×1.5^n

b) Pop B $= 400 \times 1.35^n$

	A		B
c) 3 hrs	200×1.5^3 $= 675$		400×1.35^3 $= 984$
6 hrs	200×1.5^6 $= 2278$		400×1.35^6 2421
7 hrs	200×1.5^7 $= 3417$	$>$	400×1.35^7 3268

$A > B$ after 7 hrs

5 The value of a new car is £16000. The car loses 15% of its value at the start of each year.

a Find a formula for the value of the car after n years.

b Find the value of the car after 4 years.

c After how many complete years will the car's value drop below £4000?

a) 16000×0.85^n

b) $16000 \times 0.85^4 = £8352$

c) $16000 \times 0.85^8 = £4360$

$16000 \times 0.85^9 = £3706$ ✓

After 9 years

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6 The population of a town is 52000. The population increases by 1.5% each year.

a Find the population after 6 years.

b When will the population reach 60000?

a) $52000 \times 1.015^6 = 56,859$

b) $52000 \times 1.015^{11} = 61,253$

$52000 \times 1.015^{10} = 60,348$ ✓

$52000 \times 1.015^9 = 59,456$

Reaches 60,000 after 10 years

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8 The half-life of a radioactive substance is the time it takes for the amount to go down to half of the original amount.

After how many half-lives will there be less than 1% of the radioactive substance left?

Half lives	
	100 %
1	50 %
2	25 %
3	12.5 %
4	6.25 %
5	3.125 %
6	1.5625 %
7	0.78125 %

After 7 years

Q9 *9 The table shows how the population of the world has grown since 1900.

a Draw a graph of this data.

b A growth function $P = 1.65 \times 1.0125^{(y - 1900)}$ where P is the population in billions in year y has been suggested as a model.

i Show this function on your graph.

ii What annual percentage increase in world population does the model assume?

ii) 1.25 %

Year	Population (billions)
1900	1.65
1910	1.75
1920	1.86
1930	2.07
1940	2.30
1950	2.56
1960	3.04
1970	3.71
1980	4.45
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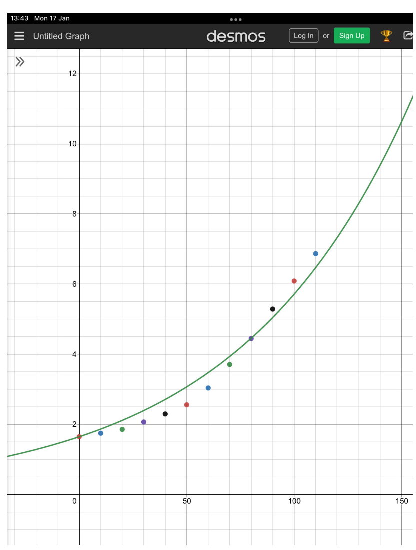

Exercise 22.5A

1 For each account in the table below, find the compound interest earned.

Acc	Original amount	Compound interest rate	Number of years
a	£250	4% per year	6
b	£840	2.5% per 6 months	5
c	£4500	1.25% per 3 months	3

2 A building society offers two accounts: Karen says that they would give the same interest on an investment. Is Karen correct? Explain your answer.

Easy Saver
4% interest added at the end of each year

Half-yearly saver
2% interest added at the end of every 6 months

3 A road planner uses the formula 2400×1.08^n to estimate the number of vehicles per day that will travel on a new road n months after it opens.

a Describe two assumptions the planner has made.

b Sketch a graph to show what the planner expects to happen.

c Give reasons why the planner's assumptions may not be appropriate.

4 There are 250 rare trees in a forest, but each year the number of trees falls by 30%. A woodland trust aims to plant 60 more trees in the forest at the end of each year.

a Show that $T_{n+1} = 0.7T_n + 60$ where T_n denotes the number of trees in the forest after n years.

b Work out the number of trees after 5 years.

c Sketch a graph to show how the number of trees varies in this time. State any assumptions you make.

5 Ben takes out a loan for £500. Interest of 2% is added to the amount owing at the end of each month, then Ben pays off £90 or all the amount owing when it is less than £90.

a How long will it take Ben to pay off the loan? Show your working.

5 b Work out the percentage interest that Ben will pay on the loan of £500.

6 Sally invests £8000 in an account that pays 3.5% interest at the end of each year. Sally has to pay 20% tax on this interest. Calculate how much Sally will have in her account at the end of 4 years.

7 Liam finds a formula for the compound interest earned by £P invested for 6 years at a rate of 4.5%. Here is Liam's method.

Interest in 1 year = 0.045 × £P
Interest for 6 years = 6 × 0.045 × £P = £0.27P

a Why is Liam's method incorrect?

b Find a correct formula.

c After 6 years the interest earned is £1934.46. Find, to the nearest one pound, the original amount £P.

8 Find the minimum rate of interest for an investment of £500 to grow to £600 in 6 years.

9 Tanya measures the temperature of a cup of coffee as it cools.

Time t (min)	0	10	20	30	40	50	60
Temperature T (°C)	85	68	55	45	39	34	31

a i Use Tanya's data to draw a graph.

ii Find the rate at which the coffee is cooling after half an hour.

b Tanya says $T = 20 + 65 \times 0.97^t$ is a good model of the data.

i Is Tanya correct? Show how you decide.

*ii Explain each term in Tanya's model.

*10 The half-life of caesium-137 is 30 years.

a Show that when 1 kilogram of caesium-137 decays, the amount left after t years is $f(t) = 2^{-\frac{t}{30}}$ kg

b Sketch a graph of amount against time.

c Describe how the function and graph would change if f(t) was given in terms of grams instead of kilograms.

1 For each account in the table below, find the compound interest earned.

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a	£250	4% per year	6
b	£840	2.5% per 6 months	5
c	£4500	1.25% per 3 months	3

1 c) $4500 \times 1.0125^{12} = £5223.40$

Interest earned £723.40

Classwork Q 1a, 1b, 2, 4

1 a) $250 \times 1.04^{6} = £316.33$

1b) $840 \times 1.025^{10} = £1075.27$

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2 A building society offers two accounts: Karen says that they would give the same interest on an investment. Is Karen correct? Explain your answer.

Easy Saver

4% interest added at the end of each year

Half-yearly saver

2% interest added at the end of every 6 months

No Annual Multipliers

Easy saver 1.04

Half-Yearly saver $1.02^2 = 1.0404$ - pays more interest

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4 There are 250 rare trees in a forest, but each year the number of trees falls by 30%. A woodland trust aims to plant 60 more trees in the forest at the end of each year.

a Show that $T_{n+1} = 0.7T_n + 60$ where T_n denotes the number of trees in the forest after n years.

b Work out the number of trees after 5 years.

c Sketch a graph to show how the number of trees varies in this time. State any assumptions you make.

a) If number falls by 30% then annual multiplier = 0.7

So $T_{n+1} = 0.7T_n$ but then 60 extra are planted

so $T_{n+1} = 0.7T_n + 60$

b) $T_0 = 250$

$T_1 = 0.7 \times 250 + 60 = 235$

$T_2 = 0.7 \times 235 + 60 = 224$

$T_3 = 0.7 \times 224 + 60 = 216$

$T_4 = 0.7 \times 216 + 60 = 211$

$T_5 = 0.7 \times 211 + 60 = 207$
