Standard Summations

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Exercise 3A

1 a) $\sum_{r=0}^{3}(2r+1) = 1+3+5+7 = 16$

b) $\sum_{r=1}^{40} r$ $\qquad \sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$

$$\sum_{r=1}^{40} r = \frac{1}{2}\times 40 \times 41 = 820$$

g) $\sum_{r=21}^{40} r = \sum_{r=1}^{40} r - \sum_{r=1}^{20} r$

$$= \frac{1}{2}\times 40\times 41 - \frac{1}{2}\times 20\times 21$$

$$= 610$$

7a) $\sum_{r=1}^{55}(3r-1) = 3\sum_{r=1}^{55} r - 55\times 1$

$\frac{3}{2}\times 55\times 56 - 55$

$= 4565$

8a Show $\sum_{r=1}^{n}(3r+2) = \frac{1}{2}n(3n+7)$

$\sum_{r=1}^{n}(3r+2) = 3\sum_{r=1}^{n} r + 2n$

$= \frac{3}{2}n(n+1) + 2n$

$= \frac{3}{2}n(n+1) + \frac{4n}{2}$

$= \frac{n}{2}\left(3(n+1)+4\right)$

$= \frac{1}{2}n(3n+7)$

Exercise 3B

1a) $\sum_{r=1}^{4} r^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$

b) $\sum_{r=1}^{40} r^2$ $\quad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

$\sum_{r=1}^{40} r^2 = \frac{40}{6}\times 41\times 81 = 22140$

2 c) Show $\sum_{r=n}^{2n} r^2 = \frac{1}{6}n(n+1)(14n+1)$

$$\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$= \frac{1}{6}(2n)(2n+1)(4n+1) - \frac{1}{6}(n-1)n(2(n-1)+1)$$

$$= \frac{1}{6}n\left[2(2n+1)(4n+1) - (n-1)(2n-1)\right]$$

$$= \frac{1}{6}n\left[2(8n^2+6n+1) - (2n^2-3n+1)\right]$$

$$= \frac{1}{6}n\left[16n^2+12n+2-2n^2+3n-1\right]$$

$$= \frac{1}{6}n\left[14n^2+15n+1\right]$$

$$= \frac{1}{6}n(n+1)(14n+1)$$

---

8) $$\sum_{r=1}^{n}(r^2+3r+1) = \sum_{r=1}^{n} r^2 + 3\sum_{r=1}^{n} r + n$$

$$= \frac{1}{6}n(n+1)(2n+1) + \frac{3}{2}n(n+1) + n$$

$$= \frac{1}{6}n(n+1)(2n+1) + \frac{9}{6}n(n+1) + \frac{6}{6}n$$

$$= \frac{1}{6}n\left[(n+1)(2n+1) + 9(n+1) + 6\right]$$

$$= \frac{1}{6}n\left[2n^2+3n+1+9n+9+6\right]$$

$$= \frac{1}{6}n\left[2n^2+12n+16\right]$$

$$= \frac{2}{6}n\left[n^2+6n+8\right]$$

$$= \frac{1}{3}n(n+2)(n+4)$$

8b $$\sum_{r=19}^{40}(r^2+3r+1) = \sum_{r=1}^{40}(r^2+3r+1) - \sum_{r=1}^{18}(r^2+3r+1)$$

$$= \frac{1}{3}\times 40\times 42\times 44 - \frac{1}{3}\times 18\times 20\times 22$$

$$= 22000$$

Classwork and Homework

Exercise 3A

Q1 d, e Q2, Q9, Q11

Exercise 3B

Q1 d, 3, 10

$(r+1)(r+3)$

(r^2+4r+3)