

Exercise 3B

(2) Show $\sum_{r=1}^n r(r+3)^2 = \frac{1}{4}n(n+1)(n^2 + an + b)$

$$\begin{aligned}
 \sum_{r=1}^n r(r+3)^2 &= \sum_{r=1}^n r(r^2 + 6r + 9) = \sum_{r=1}^n (r^3 + 6r^2 + 9r) \\
 &= \sum_{r=1}^n r^3 + 6 \sum_{r=1}^n r^2 + 9 \sum_{r=1}^n r \\
 &= \frac{1}{4}n^2(n+1)^2 + \frac{6}{6}n(n+1)(2n+1) + \frac{9}{2}n(n+1) \\
 &= \frac{1}{4}n^2(n+1)^2 + \frac{4}{4}n(n+1)(2n+1) + \frac{18}{4}n(n+1) \\
 &= \frac{1}{4}n(n+1)[n(n+1) + 4(2n+1) + 18] \\
 &= \frac{1}{4}n(n+1)[n^2 + n + 8n + 4 + 18] \\
 &= \frac{1}{4}n(n+1)(n^2 + 9n + 22)
 \end{aligned}$$

9a) $\sum_{r=1}^n r^2(r-1) = \frac{1}{12}n(n+1)(3n^2 - n - 2)$

$$\begin{aligned}
 \sum_{r=1}^{2n-1} r^2(r-1) &= \frac{1}{12}(2n-1)(2n-1+1)(3(2n-1)^2 - (2n-1)-2) \\
 &= \frac{1}{12}(2n-1)(2n)(3(4n^2 - 4n + 1) - 2n + 1 - 2) \\
 &= \frac{2}{12}(2n-1)n(12n^2 - 12n + 3 - 2n - 1) \\
 &= \frac{1}{6}(2n-1)n(12n^2 - 14n + 2)
 \end{aligned}$$

$$= \frac{1}{3}(2n-1)n(6n^2 - 7n + 1)$$

Mixed Exercise 3

$$\begin{aligned} 2(a) \quad \sum_{r=1}^n (3r-5) &= 3 \sum_{r=1}^n r - 5n \\ &= \frac{3}{2}n(n+1) - 5n \\ &= \frac{3n^2}{2} + \frac{3n}{2} - 5n \\ &= \frac{3n^2}{2} - \frac{7n}{2} \\ &= \frac{1}{2}n(3n-7) \end{aligned}$$

$$11) \quad \sum_{r=1}^n r^2 = \sum_{r=1}^{n+1} (9r+1) \quad \text{Find } n$$

$$\frac{1}{6}n(n+1)(2n+1) = 9 \sum_{r=1}^{n+1} r + n+1$$

$$\frac{1}{6}n(n+1)(2n+1) = \frac{9}{2}(n+1)(n+2) + n+1$$

$$\frac{1}{6}n(n+1)(2n+1) - \frac{27}{2}(n+1)(n+2) - \frac{6}{6}(n+1) = 0$$

$$\frac{1}{6}(n+1) \left[n(2n+1) - 27(n+2) - 6 \right] = 0$$

$$\frac{1}{6}(n+1) \left[2n^2 + n - 27n - 54 - 6 \right] = 0$$

$$\frac{1}{6}(n+1) \left[2n^2 - 26n - 60 \right] = 0$$

$$\frac{1}{3}(n+1)(n^2 - 13n - 30) = 0$$

$$\frac{1}{3}(n+1)(n+2)(n-15) = 0$$

$$n > 0 \Rightarrow n = 15$$

Continue with even numbers starting at 44
