(P) 4 Prove that:

a $\cos\theta + \sin\theta\tan\theta \equiv \sec\theta$

b $\cot\theta + \tan\theta \equiv \operatorname{cosec}\theta\sec\theta$

c $\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta$

d $(1-\cos x)(1+\sec x) \equiv \sin x\tan x$

e $\dfrac{\cos x}{1-\sin x} + \dfrac{1-\sin x}{\cos x} \equiv 2\sec x$

f $\dfrac{\cos\theta}{1+\cot\theta} \equiv \dfrac{\sin\theta}{1+\tan\theta}$

(P) 5 Solve, for values of θ in the interval $0 \leqslant \theta \leqslant 360°$, the following equations. Give your answers to 3 significant figures where necessary.

a $\sec\theta = \sqrt{2}$

b $\operatorname{cosec}\theta = -3$

c $5\cot\theta = -2$

d $\operatorname{cosec}\theta = 2$

e $3\sec^2\theta - 4 = 0$

f $5\cos\theta = 3\cot\theta$

g $\cot^2\theta - 8\tan\theta = 0$

h $2\sin\theta = \operatorname{cosec}$

(P) 6 Solve, for values of θ in the interval $-180° \leqslant \theta \leqslant 180°$, the following equations:

a $\operatorname{cosec}\theta = 1$

b $\sec\theta = -3$

c $\cot\theta = 3.45$

d $2\operatorname{cosec}^2\theta - 3\operatorname{cosec}\theta = 0$

e $\sec\theta = 2\cos\theta$

f $3\cot\theta = 2\sin\theta$

g $\operatorname{cosec}2\theta = 4$

h $2\cot^2\theta - \cot\theta - 5 = 0$

(P) 7 Solve the following equations for values of θ in the interval $0 \leqslant \theta \leqslant 2\pi$. Give your answers in terms of π.

a $\sec\theta = -1$

b $\cot\theta = -\sqrt{3}$

c $\operatorname{cosec}\frac{1}{2}\theta = \dfrac{2\sqrt{3}}{3}$

d $\sec\theta = \sqrt{2}\tan\theta \left(\theta \neq \frac{\pi}{2}, \theta \neq \frac{3\pi}{2}\right)$

f $\dfrac{\cos\theta}{1+\cot\theta} \equiv \dfrac{\sin\theta}{1+\tan\theta}$

$$\frac{\cos\theta}{1+\cot\theta} \equiv \frac{\cos\theta}{1+\frac{\cos\theta}{\sin\theta}}$$

$$\equiv \frac{\cos\theta}{1+\frac{\cos\theta}{\sin\theta}} \times \frac{\sin\theta}{\sin\theta}$$

$$\equiv \frac{\sin\theta\cos\theta}{\sin\theta+\cos\theta}$$

$$= \frac{\sin\theta\cos\theta}{\sin\theta + \cos\theta} \div \frac{\cos\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\tan\theta + 1}$$

---

5 g $\cot^2\theta - 8\tan\theta = 0$

$$\frac{\cos^2\theta}{\sin^2\theta} - \frac{8\sin\theta}{\cos\theta} = 0$$

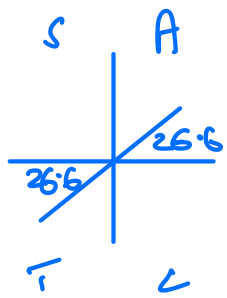
$$\frac{\cos^2\theta}{\sin^2\theta} = \frac{8\sin\theta}{\cos\theta}$$

$$\cos^3\theta = 8\sin^3\theta$$

$$\frac{1}{8} = \frac{\sin^3\theta}{\cos^3\theta} = \tan^3\theta$$

$$\Rightarrow \frac{1}{2} = \tan\theta$$

$$\theta = 26.6^\circ = 206.6^\circ$$



$\tan^{-1}\frac{1}{2} = 26.6^\circ$

6 **h** $2\cot^2\theta - \cot\theta - 5 = 0$

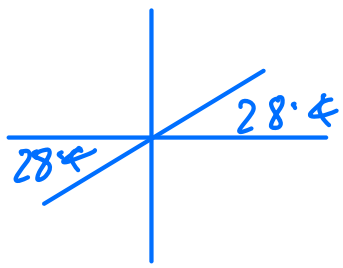
By calc

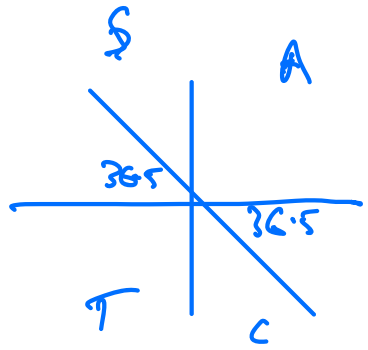
$\cot\theta = 1.85078$

or -1.35078

$\theta = \tan^{-1}\left(\frac{1}{1.85078}\right)$ or $\tan^{-1}\left(\frac{1}{-1.35078}\right)$

28.4° 36.5





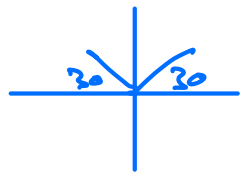
$\theta = 28.4^\circ, -151.6$ $143.5^\circ, -36.5^\circ$

Classwork Exercise 6C

4d, 4e, 5d, 5e

5 **d** $\operatorname{cosec}\theta = 2$ $\Rightarrow$ $\frac{1}{\sin\theta} = 2$

$\sin\theta = \frac{1}{2}$



$\theta = 30^\circ \qquad \theta = 150^\circ$

---

5 **e** $3\sec^2\theta - 4 = 0$

$$3\sec^2\theta = 4$$

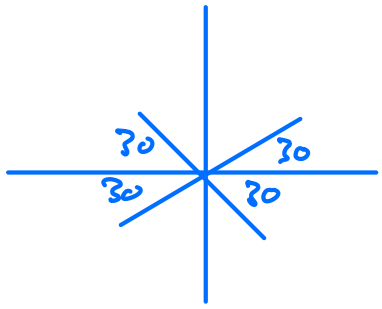
$$\sec^2\theta = \frac{4}{3}$$

$$\frac{1}{\cos^2\theta} = \frac{4}{3}$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

$\cos^{-1}\frac{\sqrt{3}}{2} = 30^\circ$



$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

---

6 **d** $2\operatorname{cosec}^2\theta - 3\operatorname{cosec}\theta = 0$

$$\operatorname{cosec}\theta(2\operatorname{cosec}\theta - 3) = 0$$

$\cancel{\operatorname{cosec}\theta = 0}$ or $2\operatorname{cosec}\theta = 3$

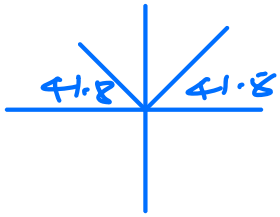
impossible

$$\operatorname{cosec}\theta = \frac{3}{2}$$

$$\sin\theta = \frac{2}{3}$$

$$\sin^{-1}\frac{2}{3} = 41.8^\circ$$

$\theta = 41.8^\circ, \quad 138.2^\circ$



---

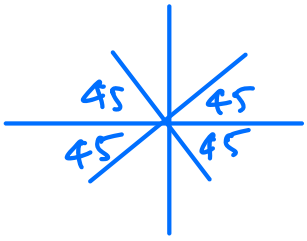
6 e $\sec\theta = 2\cos\theta$

$$\frac{1}{\cos\theta} = 2\cos\theta$$

$$\frac{1}{2} = \cos^2\theta$$

$$\cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\cos^{-1}\frac{1}{\sqrt{2}} = 45^\circ$$



$\theta = 45^\circ, 135^\circ,$
$-45^\circ, -135^\circ$

Exercise 6D

Give answers to 3 significant figures where necessary.

1 Simplify each of the following expressions.

a $1 + \tan^2 \frac{1}{2}\theta$ **b** $(\sec\theta - 1)(\sec\theta + 1)$ **c** $\tan^2\theta(\operatorname{cosec}^2\theta - 1)$

d $(\sec^2\theta - 1)\cot\theta$ **e** $(\operatorname{cosec}^2\theta - \cot^2\theta)^2$ **f** $2 - \tan^2\theta + \sec^2\theta$

g $\dfrac{\tan\theta\sec\theta}{1 + \tan^2\theta}$ **h** $(1 - \sin^2\theta)(1 + \tan^2\theta)$ **i** $\dfrac{\operatorname{cosec}\theta\cot\theta}{1 + \cot^2\theta}$

j $(\sec^4\theta - 2\sec^2\theta\tan^2\theta + \tan^4\theta)$ **k** $4\operatorname{cosec}^2 2\theta + 4\operatorname{cosec}^2 2\theta\cot^2 2\theta$

(P) **2** Given that $\operatorname{cosec} x = \dfrac{k}{\operatorname{cosec} x}$, where $k > 1$, find, in terms of k, possible values of $\cot x$.

3 Given that $\cot\theta = -\sqrt{3}$, and that $90° < \theta < 180°$, find the exact values of:

a $\sin\theta$ **b** $\cos\theta$

4 Given that $\tan\theta = \frac{3}{4}$, and that $180° < \theta < 270°$, find the exact values of:

a $\sec\theta$ **b** $\cos\theta$ **c** $\sin\theta$

5 Given that $\cos\theta = \frac{24}{25}$, and that θ is a reflex angle, find the exact values of:

a $\tan\theta$ **b** $\operatorname{cosec}\theta$

c $\tan^2\theta(\operatorname{cosec}^2\theta - 1)$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$= \tan^2\theta \cot^2\theta$

$= \tan^2\theta \times \dfrac{1}{\tan^2\theta}$

$= 1$

f $2 - \tan^2\theta + \sec^2\theta$

$= 2 - \tan^2\theta + 1 + \tan^2\theta$

$= 3$

---

i $\dfrac{\operatorname{cosec}\theta\cot\theta}{1+\cot^2\theta}$ $= \dfrac{\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta} = \dfrac{\cot\theta}{\operatorname{cosec}\theta}$

$$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$= \frac{\cos\theta}{\sin\theta} \times \frac{\sin\theta}{1}$$

$$= \cos\theta$$

---

k $4\operatorname{cosec}^2 2\theta + 4\operatorname{cosec}^2 2\theta\cot^2 2\theta$

$$4\operatorname{cosec}^2 2\theta\left(1+\cot^2 2\theta\right)$$

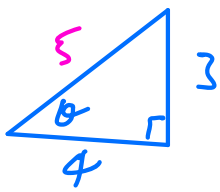
$$4\operatorname{cosec}^2 2\theta\left(\operatorname{cosec}^2 2\theta\right)$$

$$= 4\operatorname{cosec}^4 2\theta$$

---

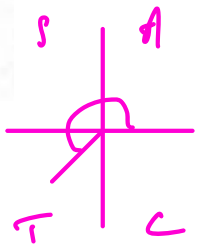
4 Given that $\tan\theta = \frac{3}{4}$, and that $180° < \theta < 270°$, find the exact values of:

a $\sec\theta$ **b** $\cos\theta$ **c** $\sin\theta$



If $\tan\theta = \frac{3}{4}$

$\sin\theta = -\frac{3}{5}$ $\cos\theta = -\frac{4}{5}$



a) $\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{4}$

b) $\cos\theta = -\frac{4}{5}$

c) $\sin\theta = -\frac{3}{5}$

Classwork Q1 left column Q3, Q5

a $1 + \tan^2 \frac{1}{2}\theta$ $= \sec^2 \frac{\theta}{2}$

d $(\sec^2\theta - 1)\cot\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$= \tan^2\theta \cot\theta$

$= \tan^2\theta \frac{1}{\tan\theta}$

$= \tan\theta$

g $\frac{\tan\theta\sec\theta}{1+\tan^2\theta}$ $= \frac{\tan\theta\sec\theta}{\sec^2\theta}$

$= \frac{\tan\theta}{\sec\theta}$

$= \frac{\sin\theta}{\cos\theta}$ $= \sin\theta$

$\frac{1}{\cos\theta}$

---

j $(\sec^4\theta - 2\sec^2\theta\tan^2\theta + \tan^4\theta)$

$= (\sec^2\theta - \tan^2\theta)^2$

$= (1 + \tan^2\theta - \tan^2\theta)^2$

$= 1^2$

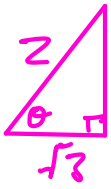
$= 1$

---

3 Given that $\cot\theta = -\sqrt{3}$, and that $90° < \theta < 180°$, find the exact values of:

a $\sin\theta$ b $\cos\theta$

$\cot\theta = -\sqrt{3} \Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$



$\sin\theta = \pm\frac{1}{2}$ $\cos\theta = \pm\frac{\sqrt{3}}{2}$

$90° < \theta < 180°$

$\Rightarrow \sin\theta = \frac{1}{2}$ $\cos\theta = -\frac{\sqrt{3}}{2}$
