

$$R \cos(\theta - \alpha)$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

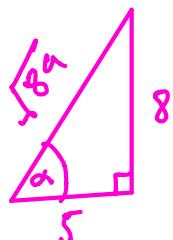
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

Ex

$$5 \sin x - 8 \cos x = 3$$



$$\sqrt{89} \left( \frac{5}{\sqrt{89}} \sin x - \frac{8}{\sqrt{89}} \cos x \right) = 3$$

$$\sqrt{89} \sin(x - \alpha)$$

$$\text{where } \tan \alpha = \frac{8}{5}$$

$$\alpha = 58.0^\circ$$

$$\sqrt{89} \sin(x - 58.0^\circ) = 3$$

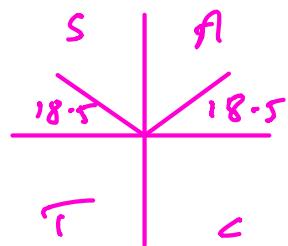
$$\sin(x - 58.0^\circ) = \frac{3}{\sqrt{89}}$$

$$\sin^{-1} \frac{3}{\sqrt{89}} = 18.5^\circ$$

$$x - 58.0^\circ = 18.5^\circ, 161.5^\circ$$

$$x = 18.5 + 58.0^\circ, 161.5 + 58.0^\circ$$

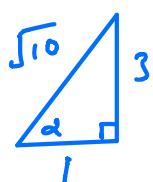
$$x = 76.5^\circ, 219.5^\circ$$



Ex 4

$$4 \sin x + 12 \cos x = 8$$

$$| \sin x + 3 \cos x | = 2$$



$$\sqrt{10} \left( \frac{1}{\sqrt{10}} \sin x + \frac{3}{\sqrt{10}} \cos x \right) = 2$$

$$\sqrt{10} \sin(x + \alpha) = 2$$

where  $\tan \alpha = \frac{3}{1}$

$$\alpha = 71.6^\circ$$

$$\sqrt{10} \sin(x + 71.6^\circ) = 2$$

$$\sin(x + 71.6) = \frac{2}{\sqrt{10}}$$

$$\sin^{-1} \frac{2}{\sqrt{10}} = 39.2^\circ$$

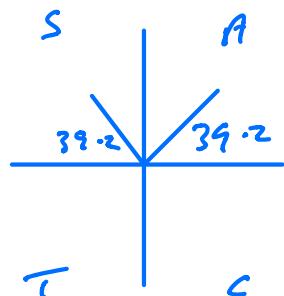
$$x + 71.6^\circ = 39.2^\circ, 140.8^\circ$$

$$x = 39.2 - 71.6, 140.8 - 71.6$$

$$x = -32.4^\circ, 69.2^\circ$$

+ 360

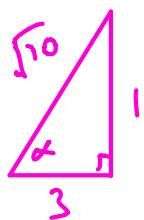
$$x = 327.6^\circ, 69.2^\circ$$



### Alternative Method

$$\sin x + 3 \cos x = 2$$

$$3 \cos x + \sin x = 2$$



$$\sqrt{10} \left( \frac{3}{\sqrt{10}} \cos x + \frac{1}{\sqrt{10}} \sin x \right) = 2$$

$$= \sqrt{10} \cos(x - \alpha) = 2$$

where  $\tan \alpha = \frac{1}{3}$

$$\alpha = 18.4^\circ$$

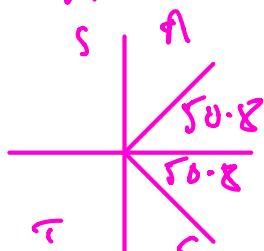
$$\sqrt{10} \cos(x - 18.4^\circ) = 2$$

$$\cos(x - 18.4) = \frac{2}{\sqrt{10}}$$

$$\cos^{-1} \frac{2}{\sqrt{10}} = 50.8^\circ$$

$$x - 18.4^\circ = 50.8^\circ, 309.2^\circ$$

$$x = 50.8 + 18.4, 309.2 + 18.4$$



$$x = 69.2^\circ, 327.6^\circ$$


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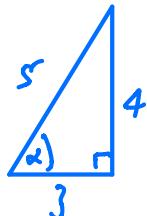
- 1 Express  $3\cos\theta + 4\sin\theta$  in the form  $R\cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ .

Hence find the range of the function  $f(\theta)$ , where

$$f(\theta) = 7 + 3\cos\theta + 4\sin\theta \quad \text{for } 0 \leq \theta \leq 2\pi.$$

Write down the greatest possible value of  $\frac{1}{7 + 3\cos\theta + 4\sin\theta}$ . [6]

$$5 \left( \frac{3}{5} \cos\theta + \frac{4}{5} \sin\theta \right)$$



$$\begin{aligned}
 &= 5 \cos(\theta - \alpha) \quad \text{where } \tan\alpha = \frac{4}{3} \\
 &\quad \alpha = 0.927 \text{ rad} \\
 &= 5 \cos(\theta - 0.927)
 \end{aligned}$$


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$$f(\theta) = 7 + 5 \cos(\theta - 0.927)$$

$$\text{Range} \quad 7-5 \leq f(\theta) \leq 7+5$$

$$2 \leq f(\theta) \leq 12$$

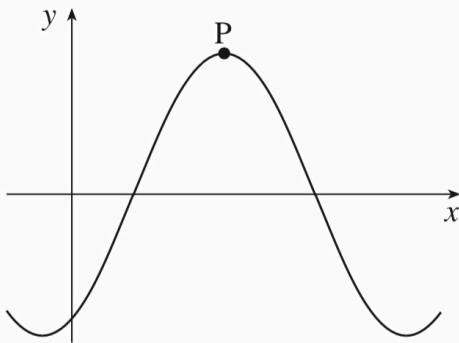

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$$\text{Max } \frac{1}{7+5 \cos(\theta - 0.927)} = \frac{1}{7-5} = \frac{1}{2}$$


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# Homework

- 1 Fig. 1 shows part of the graph of  $y = \sin x - \sqrt{3}\cos x$ .



**Fig. 1**

Express  $\sin x - \sqrt{3}\cos x$  in the form  $R \sin(x - \alpha)$ , where  $R > 0$  and  $0 \leq \alpha \leq \frac{1}{2}\pi$ .

Hence write down the exact coordinates of the turning point P.

[6]