Vectors - 2D SUVAT

2 A particle P moves with acceleration $(-3\mathbf{i} + 12\mathbf{j})\,\mathrm{m\,s^{-2}}$. Initially the velocity of P is $4\mathbf{i}\,\mathrm{m\,s^{-1}}$.

(a) Find the velocity of P at time t seconds. *(2 marks)*

(b) Find the speed of P when $t = 0.5$. *(3 marks)*

a) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} -3 \\ 12 \end{pmatrix} t \quad = (4 - 3t)\underline{i} + 12t\underline{j}$

b) When $t = 0.5$ $\quad |\underline{v}| = \sqrt{(4 - \frac{3}{2})^2 + (12 \times \frac{1}{2})^2} = 6.5\ \mathrm{m\,s^{-1}}$

6. A particle P moves with constant acceleration $(2\mathbf{i} - 5\mathbf{j})$ m s^{-2}. At time $t = 0$, P has speed u m s^{-1}. At time $t = 3$ s, P has velocity $(-6\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the value of u.

(Total 5 marks)

$$\underline{v} = \underline{u} + \underline{a}t$$

$$\begin{pmatrix} -6 \\ 1 \end{pmatrix} = \underline{u} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} \times 3$$

$$\begin{pmatrix} -6 \\ 1 \end{pmatrix} - \begin{pmatrix} 6 \\ -15 \end{pmatrix} = \underline{u}$$

$$\underline{u} = \begin{pmatrix} -12 \\ 16 \end{pmatrix} = -12\underline{i} + 16\underline{j} \text{ ms}^{-1}$$

$$|\underline{u}| = u = \sqrt{(-12)^2 + 16^2} = 20 \text{ ms}^{-1}$$

6 The points A and B have position vectors $(3\mathbf{i} + 2\mathbf{j})$ metres and $(6\mathbf{i} - 4\mathbf{j})$ metres respectively. The vectors $\mathbf{i}$ and $\mathbf{j}$ are in a horizontal plane.

(a) A particle moves from A to B with constant velocity $(\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. Calculate the time that the particle takes to move from A to B. *(3 marks)*

(b) The particle then moves from B to a point C with a constant acceleration of $2\mathbf{j}\,\mathrm{m\,s^{-2}}$. It takes 4 seconds to move from B to C.

(i) Find the position vector of C. *(4 marks)*

(ii) Find the distance AC. *(2 marks)*

a) Distance $= \sqrt{(3-6)^2 + (2--4)^2} = \sqrt{45}$ m

Speed $= \sqrt{1^2 + (-2)^2} = \sqrt{5}$ $\mathrm{ms^{-1}}$

Time $= \dfrac{\sqrt{45}}{\sqrt{5}} = \sqrt{9} = 3$ s

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b) $\underline{s} - \underline{s_0} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{s} - \begin{pmatrix} 6 \\ -4 \end{pmatrix} = 4\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 2 \end{pmatrix} \times 4^2$

$\underline{s} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} + \begin{pmatrix} 0 \\ 16 \end{pmatrix} + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} = 10\underline{i} + 4\underline{j}$

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c) $|AC| = \sqrt{(10-3)^2 + (4-2)^2}$

$= \sqrt{53}$ m

$= 7.28$ m

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8 A particle is initially at the origin, where it has velocity $(5\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. It moves with a constant acceleration $\mathbf{a}\,\mathrm{m\,s^{-2}}$ for 10 seconds to the point with position vector $75\mathbf{i}$ metres.

(a) Show that $\mathbf{a} = 0.5\mathbf{i} + 0.4\mathbf{j}$. *(3 marks)*

(b) Find the position vector of the particle 8 seconds after it has left the origin. *(3 marks)*

(c) Find the position vector of the particle when it is travelling parallel to the unit vector $\mathbf{i}$. *(6 marks)*

a)

$$\underline{s} = \underline{u}t + \tfrac{1}{2}\underline{a}t^2$$

$$\begin{pmatrix}75\\0\end{pmatrix} = \begin{pmatrix}5\\-2\end{pmatrix}t + \tfrac{1}{2}\underline{a}t^2$$

$$\begin{pmatrix}75\\0\end{pmatrix} = 10\begin{pmatrix}5\\-2\end{pmatrix} + 50\underline{a}$$

$$\begin{pmatrix}75\\0\end{pmatrix} - \begin{pmatrix}50\\-20\end{pmatrix} = 50\underline{a}$$

$$\begin{pmatrix}25\\20\end{pmatrix} = 50\underline{a}$$

$$\frac{1}{50}\begin{pmatrix}25\\20\end{pmatrix} = \underline{a} \qquad \underline{a} = 0.5\underline{i} + 0.4\underline{j}\ \mathrm{m\,s^{-2}}$$

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b)

$$\underline{s} = \underline{u}t + \tfrac{1}{2}\underline{a}t^2$$

$$\underline{s} = 8\begin{pmatrix}5\\-2\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}0.5\\0.4\end{pmatrix} \times 8^2$$

$$\underline{s} = \begin{pmatrix}40\\-16\end{pmatrix} + \begin{pmatrix}16\\12.8\end{pmatrix}$$

$$\underline{s} = \begin{pmatrix}56\\-3.2\end{pmatrix} = 56\underline{i} - 3.2\underline{j}\ \mathrm{m}$$

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c) $\underline{v} = \underline{u} + \underline{a}t$

$$\underline{v} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 0.4 \end{pmatrix} t$$

If $\underline{v}$ parallel to $\underline{i}$

$$-2 + 0.4t = 0$$

$$0.4t = 2$$

$$t = \frac{2}{0.4}$$

$$t = 5s$$

$$\underline{s} = \underline{u}t + \tfrac{1}{2}at^2$$

$$\underline{s} = 5\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \tfrac{1}{2}\begin{pmatrix} 0.5 \\ 0.4 \end{pmatrix} \times 5^2$$

$$\underline{s} = \begin{pmatrix} 25 \\ -10 \end{pmatrix} + \begin{pmatrix} 6.25 \\ 5 \end{pmatrix} = \begin{pmatrix} 31.25 \\ -5 \end{pmatrix}$$

$$= 31.25\underline{i} - 5\underline{j} \text{ m}$$

$$= 31.3\underline{i} - 5\underline{j} \text{ m}$$

8 A boat is initially at the origin, heading due east at $5\,\mathrm{m\,s^{-1}}$. It then experiences a constant acceleration of $(-0.2\mathbf{i} + 0.25\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) State the initial velocity of the boat as a vector. *(1 mark)*

(b) Find an expression for the velocity of the boat t seconds after it has started to accelerate. *(2 marks)*

(c) Find the value of t when the boat is travelling due north. *(3 marks)*

(d) Find the bearing of the boat from the origin when the boat is travelling due north. *(6 marks)*

a) $5\underline{i}\ \mathrm{ms^{-1}}$

b) $\underline{v} = \underline{u} + \underline{a}t = \begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}-0.2\\0.25\end{pmatrix}t$

$= (5 - 0.2t)\underline{i} + 0.25t\underline{j}$

c) When travelling North

$\underline{i}$ component $= 0$

$5 - 0.2t = 0$

$5 = 0.2t$

$t = 25\ \mathrm{s}$

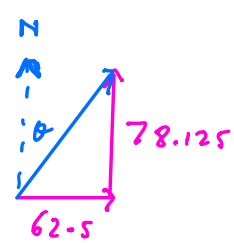
d) $\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$t = 25$

$\underline{s} = 25\begin{pmatrix}5\\0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-0.2\\0.25\end{pmatrix} \times 25^2$

$\underline{s} = \begin{pmatrix}125\\0\end{pmatrix} + \begin{pmatrix}-62.5\\78.125\end{pmatrix}$

$\underline{s} = \begin{pmatrix}62.5\\78.125\end{pmatrix}$



Bearing $\theta = \tan^{-1}\left(\frac{62.5}{78.125}\right) = 38.66°$

Bearing 038.7°

8. [*In this question, the unit vectors* **i** *and* **j** *are horizontal vectors due east and north respectively.*]

At time $t = 0$, a football player kicks a ball from the point A with position vector $(2\mathbf{i} + \mathbf{j})$ m on a horizontal football field. The motion of the ball is modelled as that of a particle moving horizontally with constant velocity $(5\mathbf{i} + 8\mathbf{j})$ m s^{-1}. Find

(a) the speed of the ball, **(2)**

(b) the position vector of the ball after t seconds. **(2)**

The point B on the field has position vector $(10\mathbf{i} + 7\mathbf{j})$ m.

(c) Find the time when the ball is due north of B. **(2)**

At time $t = 0$, another player starts running due north from B and moves with constant speed v m s^{-1}. Given that he intercepts the ball,

(d) find the value of v. **(6)**

(e) State one physical factor, other than air resistance, which would be needed in a refinement of the model of the ball's motion to make the model more realistic. **(1)**

a) speed $= \sqrt{5^2 + 8^2} = \sqrt{89} = 9.43 \text{ ms}^{-1}$

b) $\underline{s} - \underline{s_0} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{s} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} t + \frac{1}{2}\begin{pmatrix} 0 \\ 0 \end{pmatrix} t^2$

$\underline{s} = \begin{pmatrix} 5t+2 \\ 8t+1 \end{pmatrix} = (5t+2)\underline{i} + (8t+1)\underline{j}$ m

c) If ball due North of B then ball and B have same $\underline{i}$ component

$\Rightarrow 5t + 2 = 10$

$5t = 8$

$t = 1.6$ s

d) When $t = 2.4$, position of ball

$$\underline{s} = \begin{pmatrix} 5(1.6)+2 \\ 8(1.6)+1 \end{pmatrix} = \begin{pmatrix} 10 \\ 13.8 \end{pmatrix}$$

Runner started at $\begin{pmatrix} 10 \\ 7 \end{pmatrix}$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{13.8 - 7}{1.6} = 4.25 \text{ ms}^{-1}$$

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e) friction or spin

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6. [*In this question the horizontal unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A model boat A moves on a lake with constant velocity $(-\mathbf{i} + 6\mathbf{j})$ m s^{-1}. At time $t = 0$, A is at the point with position vector $(2\mathbf{i} - 10\mathbf{j})$ m. Find

(a) the speed of A, **(2)**

(b) the direction in which A is moving, giving your answer as a bearing. **(3)**

At time $t = 0$, a second boat B is at the point with position vector $(-26\mathbf{i} + 4\mathbf{j})$ m.

Given that the velocity of B is $(3\mathbf{i} + 4\mathbf{j})$ m s^{-1},

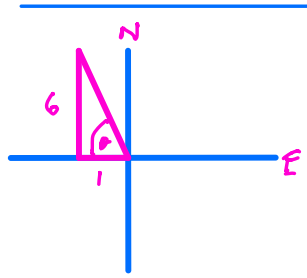
(c) show that A and B will collide at a point P and find the position vector of P. **(5)**

Given instead that B has speed 8 m s^{-1} and moves in the direction of the vector $(3\mathbf{i} + 4\mathbf{j})$,

(d) find the distance of B from P when $t = 7$ s. **(6)**

a) speed $= \sqrt{(-1)^2 + 6^2} = \sqrt{37} = 6.08$ m s^{-1}

b)



$\theta = \tan^{-1}\left(\frac{6}{1}\right) = 80.54°$

Bearing $= 270 + 80.54°$

$= 351°$

c) For A

$\underline{s} - \underline{s}_0 = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{s} - \begin{pmatrix} 2 \\ -10 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} t + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\underline{s}_A = \begin{pmatrix} 2 - t \\ -10 + 6t \end{pmatrix}$

For B

$\underline{s} - \underline{s}_0 = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$\underline{s} - \begin{pmatrix} -26 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} t + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\underline{s}_B = \begin{pmatrix} -26 + 3t \\ 4 + 4t \end{pmatrix}$$

If $\underline{s}_A = \underline{s}_B$

$$2 - t = -26 + 3t$$
$$2 + 26 = 3t + t$$
$$28 = 4t$$
$$t = 7s$$

$\underline{i}$ components match at $t = 7s$

Also

$$-10 + 6t = 4 + 4t$$
$$6t - 4t = 4 + 10$$
$$2t = 14$$
$$t = 7s$$

$\underline{j}$ components match at $t = 7s$

$\therefore$ A and B collide when $t = 7s$

when $t = 7$

$$\begin{pmatrix} 2 - t \\ -10 + 6t \end{pmatrix} = \begin{pmatrix} 2 - 7 \\ -10 + 6(7) \end{pmatrix} = P \begin{pmatrix} -5 \\ 32 \end{pmatrix}$$

$$P = -5\underline{i} + 32\underline{j}$$

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d) unit vector in direction $(3\underline{i} + 4\underline{j})$

$$= \frac{1}{\sqrt{3^2 + 4^2}}(3\underline{i} + 4\underline{j})$$
$$= \frac{1}{5}(3\underline{i} + 4\underline{j})$$

$\therefore$ B has velocity $\frac{8}{5}(3\underline{i} + 4\underline{j}) = \begin{pmatrix} \frac{24}{5} \\ \frac{32}{5} \end{pmatrix}$

For B $\quad \underline{s}_B - \underline{s}_0 = \underline{u}t + \frac{1}{2}at^2$

$$\underline{s}_B - \begin{pmatrix} -26 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{24}{5} \\ \frac{32}{5} \end{pmatrix} t + \frac{1}{2} \underline{a} t^2$$

$$\underline{s}_B = \begin{pmatrix} -26 + \frac{24}{5} t \\ 4 + \frac{32}{5} t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

When $t = 7s$

$$\underline{s}_B = \begin{pmatrix} -26 + \frac{24}{5}(7) \\ 4 + \frac{32}{5}(7) \end{pmatrix} = \begin{pmatrix} 7.6 \\ 48.8 \end{pmatrix}$$

$$P = \begin{pmatrix} -5 \\ 32 \end{pmatrix}$$

Distance of B from P

$$= \sqrt{(7.6 - -5)^2 + (48.8 - 32)^2}$$

$$= 21 \text{ m}$$