

# Hypothesis Testing

3 Over a long period of time, 20% of all bowls made by a particular manufacturer are imperfect and cannot be sold.

- (i) Find the probability that fewer than 4 bowls from a random sample of 10 made by the manufacturer are imperfect. [2]

The manufacturer introduces a new process for producing bowls. To test whether there has been an improvement, each of a random sample of 20 bowls made by the new process is examined. From this sample, 2 bowls are found to be imperfect.

- (ii) Show that this does not provide evidence, at the 5% level of significance, of a reduction in the proportion of imperfect bowls. You should show your hypotheses and calculations clearly. [6]

i) Let  $X$  be number of imperfect bowls

$$X \sim B(10, 0.2)$$

$$P(X < 4) = P(X \leq 3) = 0.8791$$

---

ii)  $E(X) = np = 20 \times 0.2 = 4$

$$H_0: p = 0.2$$

$$H_1: p < 0.2$$

where  $p$  is the prob a randomly chosen bowl is imperfect

$$X \sim B(20, 0.2)$$

$$P(X \leq 2) = 0.2061 > 5\%$$

Accept  $H_0$

There is not sufficient evidence to support the view the process has improved. Accept 20%

imperfect.

F14

5

- 7 A geologist splits rocks to look for fossils. On average 10% of the rocks selected from a particular area do in fact contain fossils.

The geologist selects a random sample of 20 rocks from this area.

(i) Find the probability that

(A) exactly one of the rocks contains fossils, [3]

(B) at least one of the rocks contains fossils. [3]

(ii) A random sample of  $n$  rocks is selected from this area. The geologist wants to have a probability of 0.8 or greater of finding fossils in at least one of the  $n$  rocks. Find the least possible value of  $n$ . [3]

(iii) The geologist explores a new area in which it is claimed that less than 10% of rocks contain fossils. In order to investigate the claim, a random sample of 30 rocks from this area is selected, and the number which contain fossils is recorded. A hypothesis test is carried out at the 5% level.

(A) Write down suitable hypotheses for the test. [3]

(B) Show that the critical region consists only of the value 0. [4]

(C) In fact, 2 of the 30 rocks in the sample contain fossils. Complete the test, stating your conclusions clearly. [2]

Let  $X$  be prob a rock contains a fossil

$$X \sim B(20, 0.1)$$

$$i) A) P(X=1) = 0.2701$$

$$\begin{aligned} B) P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - 0.1^{20} \\ &= 0.8784 \end{aligned}$$

$$\text{ii) } n=16 \quad P(X \geq 1) = 1 - 0.9^{16} = 0.8147 > 0.8$$

$$n=15 \quad P(X \geq 1) = 1 - 0.9^{15} = 0.7941 < 0.8$$

$$\underline{n = 16}$$

$$\text{iii) A) } H_0 : p = 0.1 \quad \text{where } p \text{ is prob a rock contains a fiss.}$$

$$H_1 : p < 0.1$$

$$\text{B) } P(X=0) = 0.9^{30} = 0.0423 < 5\%$$

$$P(X=1) = \binom{30}{1} \times 0.1^1 \times 0.9^{29} = 0.1413$$

$$\therefore P(X \leq 1) = 0.1836 > 5\%$$

$$\therefore \text{critical region} = \{0\}$$

$$\text{C) } P(X=2) = \binom{30}{2} \times 0.1^2 \times 0.9^{28}$$

$$= 0.2277$$

$$P(X \leq 2) = 0.2277 + 0.1413 + 0.0423$$

$$= 0.4113 > 5\%$$

Accept  $H_0$

There is not sufficient evidence to support the view the proportion has decreased  
Accept proportion is 10%

