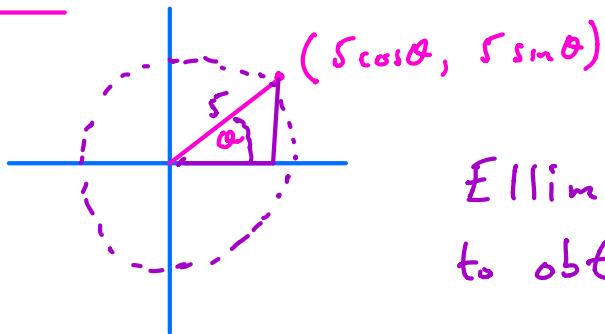


## Parametric Equations

Ex1  $x = 5 \cos \theta$        $y = 5 \sin \theta$        $0 \leq \theta \leq 2\pi$

Circle



Eliminating the parameter  
to obtain the cartesian eqn

$$x = 5 \cos \theta \quad y = 5 \sin \theta$$

$$x^2 + y^2 = 5^2 \cos^2 \theta + 5^2 \sin^2 \theta$$

$$x^2 + y^2 = 5^2 (\cos^2 \theta + \sin^2 \theta)$$

$$\underline{x^2 + y^2 = 5^2}$$

Find the gradient  $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = \frac{dy}{d\theta} / \cancel{\frac{d\theta}{dx}}$$

$$x = 5 \cos \theta \quad y = 5 \sin \theta$$

$$\frac{dx}{d\theta} = -5 \sin \theta \quad \frac{dy}{d\theta} = 5 \cos \theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} / \cancel{\frac{d\theta}{dx}} = \frac{5 \cos \theta}{-5 \sin \theta} = -\cot \theta$$

Why find the gradient?

Ex Find eqn of tangent at any point on circle with parameter  $\theta$

General point  $(5\cos\theta, 5\sin\theta)$

$$\text{Tgt } y - y_1 = m(x - x_1)$$

$$y - 5\sin\theta = -\cot\theta(x - 5\cos\theta)$$

$$y = -\cot\theta x + 5\cos\theta \cot\theta + 5\sin\theta$$

---

Find tgt when  $\theta = \frac{\pi}{3}$

$$y = -\frac{1}{\sqrt{3}}x + 5 \times \frac{1}{2} \times \frac{1}{\sqrt{3}} + 5 + \frac{5\sqrt{3}}{2}$$

$$y = -\frac{1}{\sqrt{3}}x + \frac{5}{2\sqrt{3}} + \frac{5\sqrt{3}}{2}$$

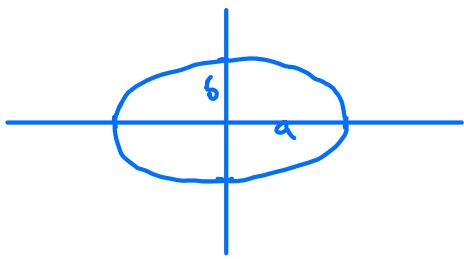
$$2\sqrt{3}y = -2x + 5 + 15$$

$$2x + 2\sqrt{3}y = 20$$

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## Ellipse

$$x = a \cos \theta, \quad y = b \sin \theta$$



$$\frac{x}{a} = \cos \theta \quad \frac{y}{b} = \sin \theta$$

$$\frac{x^2}{a^2} = \cos^2 \theta \quad \frac{y^2}{b^2} = \sin^2 \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 \theta + \sin^2 \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$


---

Ex3

$$\frac{(x-3)^2}{5^2} + \frac{(y-2)^2}{4^2} = 1$$

Ellipse centred on (3, 2)

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Parabola

$$x = at^2, \quad y = 2at$$

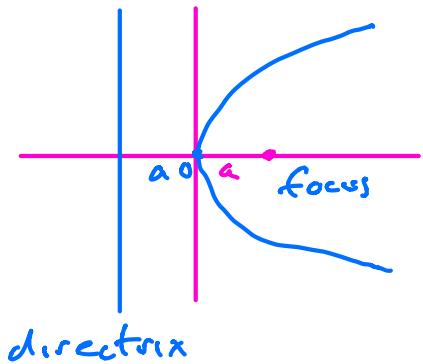
Eliminate t

$$t = \frac{y}{2a}$$

Sub for t       $x = a \left( \frac{y}{2a} \right)^2$

$$x = \frac{y^2}{4a}$$

$$\underline{y^2 = 4ax}$$



Every point on parabola  
is equidistant from the focus  $(a, 0)$   
and the directrix line  $x = -a$   
(Do not need to know this)

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$$\text{Find } \frac{dy}{dx} \quad x = at^2 \quad y = 2at \quad t \in \mathbb{R}$$

$$\frac{dx}{dt} = 2at \quad \frac{dy}{dt} = 2a$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Find eqn of tgt at general point  $(at^2, 2at)$

$$y - y_1 = m(x - x_1)$$

$$y - 2at = \frac{1}{t}(x - at^2)$$

$$y = \frac{1}{t}x - at + 2at$$

$$\underline{y = \frac{1}{t}x + at}$$

$$\text{or } \underline{ty = x + at^2}$$
$$\underline{x - ty + at^2 = 0}$$

## Trigonometric Essentials

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \begin{cases} \cos^2 A - \sin^2 A \\ 1 - 2 \sin^2 A \\ 2 \cos^2 A - 1 \end{cases}$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

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### Exercise 9A

1 ii  $x = \cos 2\theta = 1 - 2 \sin^2 \theta$

$$y = \sin^2 \theta$$

$$\Rightarrow x = 1 - 2y^2$$

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1 v)  $x = 2 \operatorname{cosec} \theta$

$$y = 2 \cot \theta$$

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$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

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