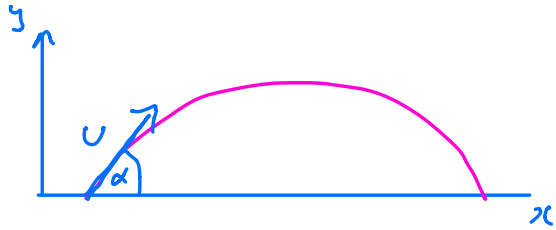
Projectiles - Find the Cartesian Equation of the Path



$$x = (U\cos\alpha)t \qquad (1)$$

$$y = (U\sin\alpha)t - \frac{1}{2}gt^2 \qquad (2)$$

From (1) $\quad t = \dfrac{x}{U\cos\alpha}$

Sub for t in (2)

$$y = \frac{(U\sin\alpha)\,x}{U\cos\alpha} - \frac{1}{2}g\frac{x^2}{U^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{gx^2}{2U^2}\sec^2\alpha$$

$$y = x\tan\alpha - \frac{gx^2}{2U^2}\left(1+\tan^2\alpha\right)$$

2 A ball is kicked from ground level over horizontal ground. It leaves the ground at a speed of $25\,\mathrm{m\,s^{-1}}$ and at an angle θ to the horizontal such that $\cos\theta = 0.96$ and $\sin\theta = 0.28$.

(i) Show that the height, y m, of the ball above the ground t seconds after projection is given by $y = 7t - 4.9t^2$. Show also that the horizontal distance, x m, travelled by this time is given by $x = 24t$. [3]

(ii) Calculate the maximum height reached by the ball. [2]

(iii) Calculate the times at which the ball is at half its maximum height.

Find the horizontal distance travelled by the ball between these times. [4]

(iv) Determine the following when $t = 1.25$.

(A) The vertical component of the velocity of the ball.

(B) Whether the ball is rising or falling. (You should give a reason for your answer.)

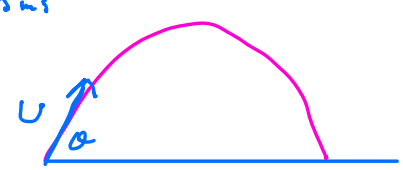
(C) The speed of the ball. [5]

(v) Show that the equation of the trajectory of the ball is

$$y = \frac{0.7x}{576}(240 - 7x).$$

Hence, or otherwise, find the range of the ball. [5]

$U = 25\,\mathrm{ms^{-1}}$



$\cos\theta = 0.96$
$\sin\theta = 0.28$

i) $x = U\cos\theta \times t = 25 \times 0.96t = 24t$

$y = U\sin\theta \times t - \frac{1}{2}gt^2$
$= 25 \times 0.28t - 4.9t^2$
$= 7t - 4.9t^2$

ii) $V_y^2 = U_y^2 + 2as$

$V_y^2 = 7^2 - 19.6y$

At max height $V_y = 0$

$$0 = 7^2 - 19.6y$$

$$19.6y = 49$$

$$y = \frac{49}{19.6} = 2.5 \text{ m}$$

iii) Half max height $= 2.5 \div 2 = 1.25$ m

$$y = 7t - 4.9t^2$$

$$1.25 = 7t - 4.9t^2$$

$$4.9t^2 - 7t + 1.25 = 0$$

By calc $t = 1.219$ s $\quad t = 0.2092$ s

$t = 1.22$ s $\quad t = 0.21$ s

when $t = 1.219$ $\quad x = 24 \times 1.219 = 29.256$ m

$t = 0.2092$ $\quad x = 24 \times 0.2092 = 5.0208$ m

Distance travelled $= 29.256 - 5.0208$

$= 24.2$ m

iv) $t = 1.25$

A) $v_y = u_y + at$

$v_y = 7 - 9.8t$

$v_y = 7 - 9.8 \times 1.25$

$v_y = -5.25 \text{ ms}^{-1}$

B) Falling since $v_y < 0$

c) $\text{Speed} = \sqrt{V_x^2 + V_y^2}$

$V_{xc} = V_x = 24$

$\text{speed} = \sqrt{24^2 + (-5.25)^2} = 24.6\ ms^{-1}$

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v) $x = 24t$ ①

$y = 7t - 4.9t^2$ ②

From ① $t = \frac{x}{24}$

Sub in ② $y = \frac{7x}{24} - 4.9\frac{x^2}{576}$

$y = \frac{168x - 4.9x^2}{576}$

$y = \frac{0.7x(240 - 7x)}{576}$

Lands when $y = 0$

$\Rightarrow \frac{0.7x(240-7x)}{576} = 0$

$\Rightarrow x = 0$ or $240 - 7x = 0$

$240 = 7x$

$\frac{240}{7} = x$

$x = 34.3\ m$

Range $= 34.3\ m$