Integration by Parts

$$\int_2^3 x^2 e^{4x}\,dx$$

Let $u = x^2$ $\Rightarrow \frac{du}{dx} = 2x$

Let $\frac{dv}{dx} = e^{4x}$ $\Rightarrow v = \frac{1}{4}e^{4x}$

$$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$$

$$\int_2^3 x^2 e^{4x}\,dx = \left[\frac{1}{4}x^2 e^{4x}\right]_2^3 - \int_2^3 \frac{1}{2}x e^{4x}\,dx \qquad (*)$$

Aside $\left[\int_2^3 \frac{1}{2}x e^{4x}\,dx\right.$

Let $u = \frac{1}{2}x$ $\Rightarrow \frac{du}{dx} = \frac{1}{2}$

$\frac{dv}{dx} = e^{4x}$ $v = \frac{1}{4}e^{4x}$

$$\int_2^3 \frac{1}{2}x e^{4x}\,dx = \left[\frac{1}{8}x e^{4x}\right]_2^3 - \int_2^3 \frac{1}{8}e^{4x}\,dx$$

$$\left.= \left[\frac{1}{8}x e^{4x}\right]_2^3 - \left[\frac{1}{32}e^{4x}\right]_2^3\right]$$

Sub into $(*)$

$$\int_2^3 x^2 e^{4x}\,dx = \left[\frac{1}{4}x^2 e^{4x}\right]_2^3 - \left[\frac{1}{8}x e^{4x}\right]_2^3 + \left[\frac{1}{32}e^{4x}\right]_2^3$$

$$\frac{d}{dx} e^{x^2}$$

$$\int e^{x^2}\,dx$$

$$2x e^{x^2}$$

$$\int_0^1 5x e^{x^2}\,dx$$

Let $u = x^2$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\frac{5}{2}\,du = 5x\,dx$$

$$= \int_0^1 \frac{5}{2} e^u\,du$$

$$= \left[\frac{5}{2} e^u\right]_0^1$$

$$\frac{5}{2}(e - 1)$$

$$\cos^4 x$$

$$y = (\cos x)^4$$

$$\frac{dy}{dx} = 4(\cos x)^3(-\sin x)$$