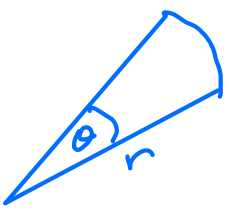
Circular Measure

Degrees	Radians
360°	2π
180°	π
90°	$\frac{\pi}{2}$
45°	$\frac{\pi}{4}$
60°	$\frac{\pi}{3}$
30°	$\frac{\pi}{6}$
120°	$\frac{2\pi}{3}$
150°	$\frac{5\pi}{6}$
135°	$\frac{3\pi}{4}$

1 radian $\approx 57.3^\circ$

Arc Length

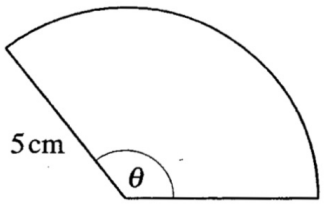


Arc length $= r\theta$

Area of Sector

Area of Sector $= \frac{1}{2}r^2\theta$

7



Not to scale

Fig. 7

Fig. 7 shows a sector of a circle of radius 5 cm which has angle θ radians. The sector has area 30 cm^2.

(i) Find θ. [3]

(ii) Hence find the perimeter of the sector. [2]

i) Area of sector $= \frac{1}{2} r^2 \theta$

$30 = \frac{1}{2} \times 5^2 \theta$

$30 = 12.5\theta$

$\frac{30}{12.5} = \theta$

$\theta = 2.4$ radians

ii) Perimeter = Arc + two radii

$= r\theta + r + r$

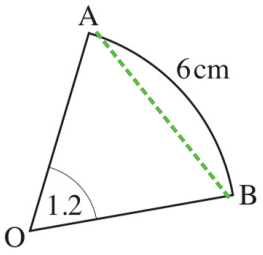
$= 5 \times 2.4 + 5 + 5$

$= 22$ cm

7 In Fig. 7, A and B are points on the circumference of a circle with centre O.

Angle AOB = 1.2 radians.

The arc length AB is 6 cm.



Not to scale

Fig. 7

(i) Calculate the radius of the circle. [2]

(ii) Calculate the length of the chord AB. [3]

i) Arc Length $= r\theta$

$6 = 1.2r$

$\frac{6}{1.2} = r$

$r = 5\text{ cm}$

ii) Cosine Rule $AB^2 = r^2 + r^2 - 2 \times r \times r \cos 1.2$

$AB^2 = 5^2 + 5^2 - 50\cos 1.2$

$AB^2 = 31.88$

$AB = \sqrt{31.88} = 5.65\text{ cm}$

7

Not to scale

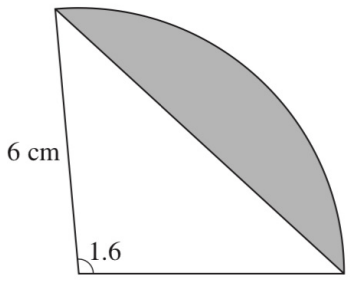


Fig. 7

A sector of a circle of radius 6 cm has angle 1.6 radians, as shown in Fig. 7.

Find the area of the sector.

Hence find the area of the shaded segment. [5]

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 6^2 \times 1.6$

$= 28.8\text{cm}^2$

Area of Segment = Area of sector − Area of Triangle

$= 28.8 - \frac{1}{2} r \times r \sin 1.6$

$= 28.8 - \frac{1}{2} \times 6 \times 6 \times \sin 1.6$

$= 10.8\text{ cm}^2$

Conversions Degrees ⟷ Radians

To convert Degrees to Radians $\times \frac{\pi}{180}$

To convert Radians to Degrees $\times \frac{180}{\pi}$

Examples

Convert $\frac{7\pi}{6}$ radians to degrees

$$\frac{7\pi}{6} \times \frac{180}{\pi} = 210^\circ$$

Convert 240° into radians

$$240 \times \frac{\pi}{180} = \frac{4\pi}{3}$$

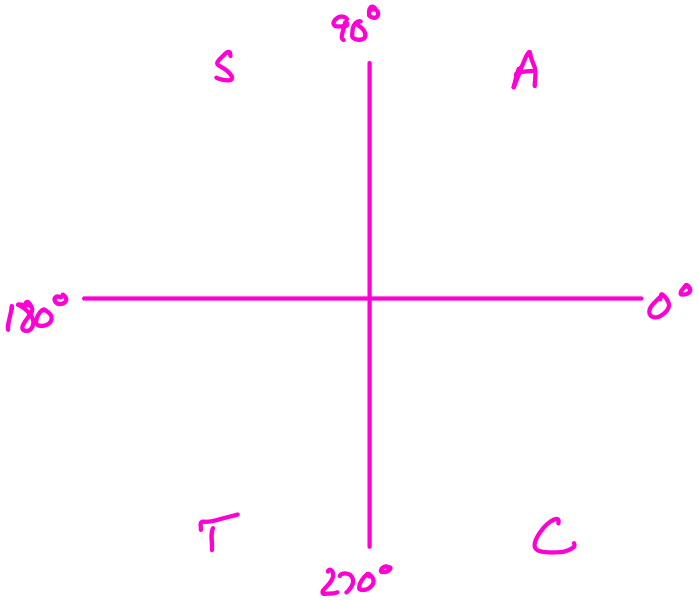
Convert 37° into radians

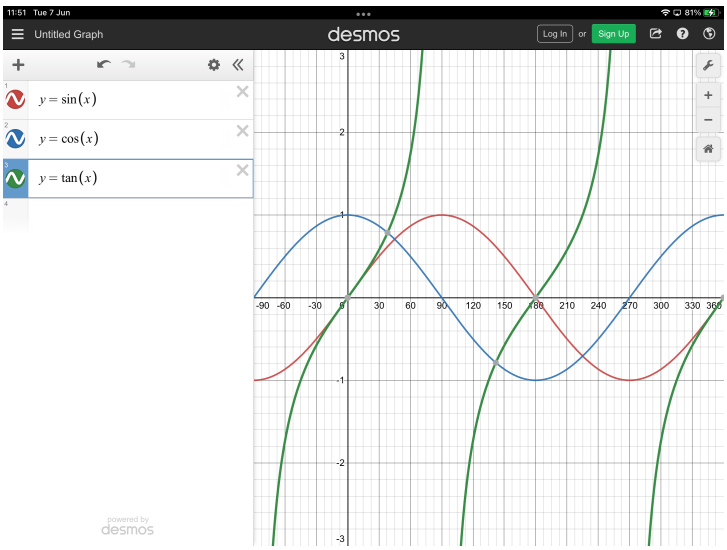
$$37 \times \frac{\pi}{180} = 0.646 \text{ radians}$$

Convert 4 radians into degrees

$$4 \times \frac{180}{\pi} = 229.18^\circ$$

CAST Diagram



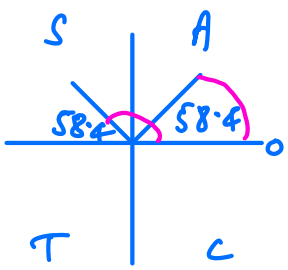


Solving simple trigonometric equations

Solve for $0 \leq \theta < 360°$

Ex 1 $\sin x = 0.852$

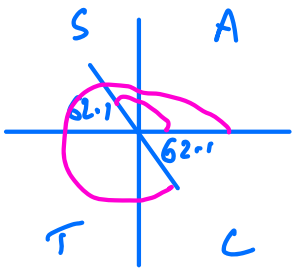
$\sin^{-1} 0.852 = 58.4°$



$\theta = 58.4°$

$\theta = 180 - 58.4 = 121.6°$

Ex 2 Solve $\tan\theta = -1.888$

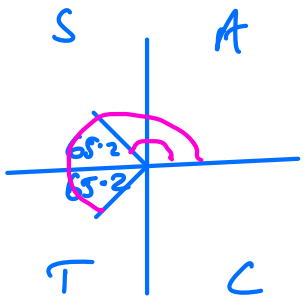


$\tan^{-1} 1.888 = 62.1°$

$\theta = 180 - 62.1 = 117.9^\circ$

$\theta = 360 - 62.1 = 297.9^\circ$

Ex 3 Solve $\cos\theta = -0.42$



$\cos^{-1} 0.42$

$= 65.2^\circ$

$\theta = 180 - 65.2 = 114.8^\circ$

$\theta = 180 + 65.2 = 245.2^\circ$

Exercise Solve for $0 \leq \theta < 360^\circ$

1) $\sin\theta = 0.68$

2) $\cos\theta = -0.89$

3) $\tan\theta = 5.1$

4) $\sin\theta = -0.222$

5) $\cos\theta = 0.111$

6) $\tan\theta = -2.4$
