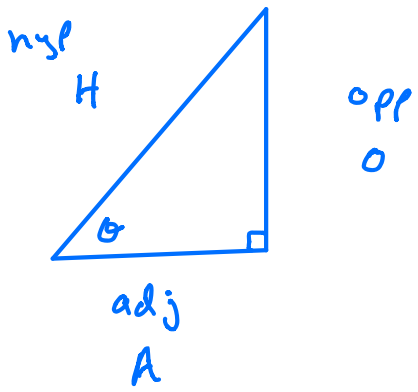
Trigonometric Identities



$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

Consider $\sin^2\theta + \cos^2\theta$

$$= \frac{O^2}{H^2} + \frac{A^2}{H^2} = \frac{O^2 + A^2}{H^2} = \frac{H^2}{H^2} = 1$$

$$\boxed{\text{Identity} \qquad \cos^2\theta + \sin^2\theta = 1}$$

True for any angle θ, any size, and also negative

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{O}{H}}{\frac{A}{H}} = \frac{O}{H} \times \frac{H}{A} = \frac{O}{A} = \tan\theta$$

$$\boxed{\text{Identity} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}}$$

4. (a) Show that the equation

$$3\sin^2\theta - 2\cos^2\theta = 1$$

can be written as

$$5\sin^2\theta = 3.$$

(2)

(b) Hence solve, for $0° \leqslant \theta < 360°$, the equation

$$3\sin^2\theta - 2\cos^2\theta = 1,$$

giving your answers to 1 decimal place.

(7)

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a)

$$3\sin^2\theta - 2\cos^2\theta = 1$$

$$3\sin^2\theta - 2(1-\sin^2\theta) = 1$$

$$3\sin^2\theta - 2 + 2\sin^2\theta = 1$$

$$5\sin^2\theta = 1+2$$

$$\underline{5\sin^2\theta = 3}$$

b)

$$\sin^2\theta = \frac{3}{5}$$

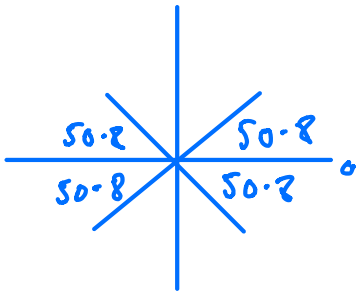
$$\sin\theta = \pm\sqrt{\frac{3}{5}}$$

$$\theta = 50.8°$$

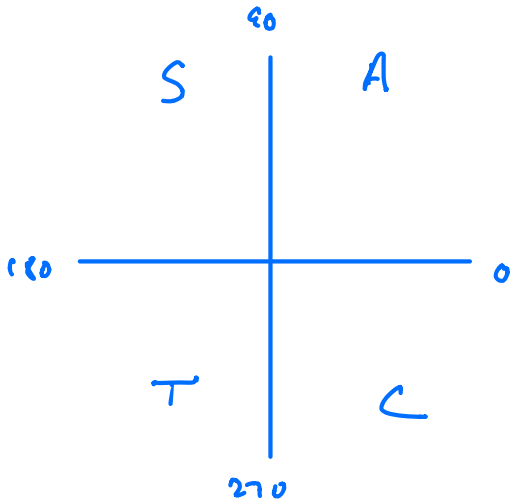
$$= 129.2°$$

$$= 230.8°$$

$$= 309.2°$$



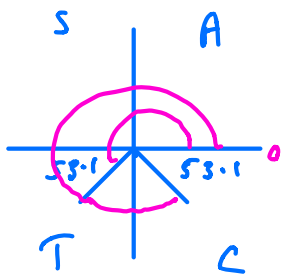
CAST Diagram



Ex 1 for $0^\circ \le \theta \le 360^\circ$

Solve $\sin\theta = -0.8$

$\sin^{-1}(0.8) = 53.1^\circ$

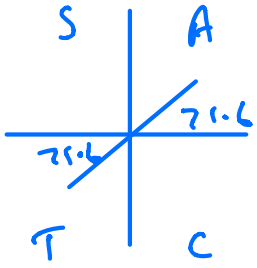


$\theta = 233.1^\circ$

$\theta = 306.9^\circ$

Ex 2 Solve $\tan\theta = 3$

$\tan^{-1} 3 = 71.6^\circ$

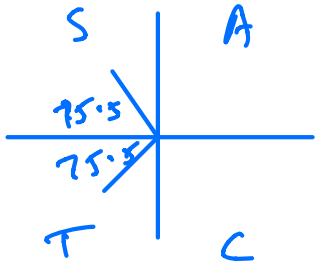


$\theta = 71.6^\circ$

$\theta = 251.6^\circ$

Ex 3 Solve $\cos\theta = -0.25$

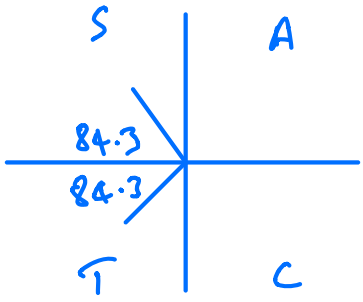
$\cos^{-1} 0.25 = 75.5^\circ$



$\theta = 104.5°$

$\theta = 255.5°$

Ex 4 $\cos 2\theta = -0.1$ for $0 \leq \theta \leq 360$



$\cos^{-1} 0.1 = 84.3°$

$2\theta = 95.7 \quad 264.3, 455.7, 624.3$

$\theta = 47.85, 182.15, 227.85, 312.15$

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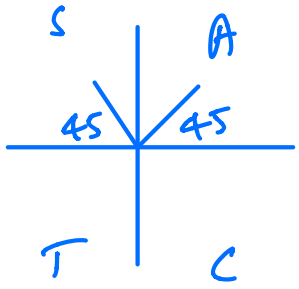
9. Solve, for $0 \leqslant x < 360°$,

(a) $\sin(x - 20°) = \dfrac{1}{\sqrt{2}}$ **(4)**

(b) $\cos 3x = -\dfrac{1}{2}$ **(6)**

a) $\sin(x - 20) = \dfrac{1}{\sqrt{2}}$ $\sin^{-1} \dfrac{1}{\sqrt{2}} = 45°$



$x - 20 = 45°, 135°$

$x = 65°, 155°$

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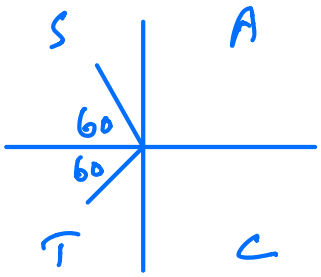
b) $\cos 3x = -\frac{1}{2}$ $\qquad$ $\cos^{-1}\frac{1}{2} = 60^\circ$



$3x = 120, 240, 480, 600, 840, 960$

$x = 40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ, 320^\circ$

Hwk $\quad$ Use CAST Diagram to solve for $0 \le \theta \le 360^\circ$

1) $\sin\theta = -0.7$

2) $\cos\theta = 0.4$

3) $\tan\theta = -2.6$

4) $\sin\theta = 0.15$

5) $\cos\theta = -0.35$

6) $\tan\theta = 1.3$

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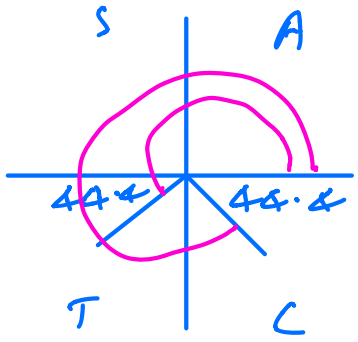
1) $\sin\theta = -0.7$

$\sin^{-1} 0.7 = 44.4$

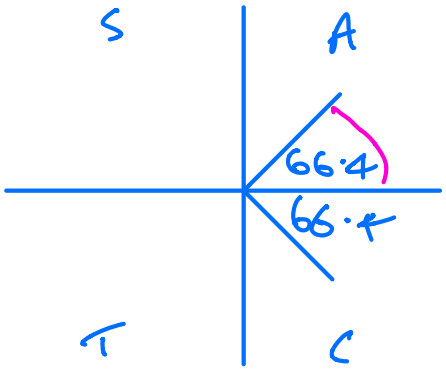


$\theta = 224.4^\circ$

$\theta = 315.6^\circ$

2) $\cos\theta = 0.4$

$\cos^{-1} 0.4 = 66.4^\circ$

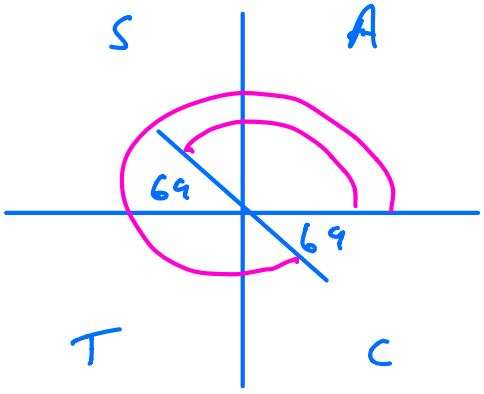


$\theta = 66.4^\circ$

$\theta = 293.6^\circ$

$\tan\theta = -2.6$

$\tan^{-1} 2.6 = 69.0^\circ$



$\theta = 111^\circ$

$\theta = 291^\circ$