Exercise 2E   Loci
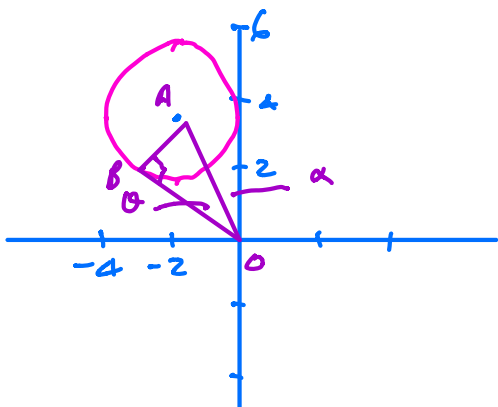
$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

5) $|z + 2 - 2\sqrt{3}i| = 2$

a) $|z - (-2 + 2\sqrt{3}i)| = 2$

Circle centre $-2 + 2\sqrt{3}i$

radius 2

b)   min $\arg z = \dfrac{\pi}{2}$

c) $|OA| = \sqrt{(-2)^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = 4$

$\theta = \sin^{-1}\dfrac{AB}{OA} = \sin^{-1}\dfrac{2}{4} = \dfrac{\pi}{6}$

$\alpha = \sin^{-1}\dfrac{2}{4} = \dfrac{\pi}{6}$

max $\arg z = \dfrac{\pi}{2} + \alpha + \theta = \dfrac{\pi}{2} + \dfrac{\pi}{6} + \dfrac{\pi}{6}$
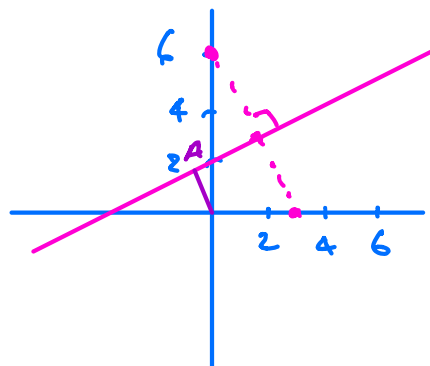
$= \dfrac{5\pi}{6}$

7) $|z - 3| = |z - 6i|$

Midpoint $\left(\dfrac{3}{2}, 3\right)$

Line has gradient $= +\dfrac{1}{2}$

$y - y_1 = m(x - x_1)$

$$y - 3 = \frac{1}{2}\left(x - \frac{3}{2}\right)$$

$$y - 3 = \frac{1}{2}x - \frac{3}{4}$$

$$\underline{y = \frac{1}{2}x + \frac{9}{4}}$$

OA given by $\underline{y = -2x}$

Find A

$$-2x = \frac{1}{2}x + \frac{9}{4}$$

$$-\frac{5x}{2} = \frac{9}{4}$$

$$-5x = \frac{18}{4}$$

$$x = -\frac{18}{20} = -\frac{9}{10}$$

$$y = -2\left(-\frac{9}{10}\right) = \frac{9}{5}$$

$$\therefore A\left(-\frac{9}{10}, \frac{9}{5}\right)$$

$$\text{Min } |z| = \sqrt{\left(-\frac{9}{10}\right)^2 + \left(\frac{18}{10}\right)^2} = \sqrt{\frac{405}{100}}$$

$$= \frac{9\sqrt{5}}{10}$$

Exercise 2E Even Numbers 4 onwards