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**1.** $$f(x) = (2 - 5x)^{-2}, \qquad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f($x$), in ascending powers of $x$, as far as the term in $x^3$, giving each coefficient as a simplified fraction.

**(5)**

$$(2-5x)^{-2} = \left(2\left(1-\tfrac{5}{2}x\right)\right)^{-2} = 2^{-2}\left(1-\tfrac{5}{2}x\right)^{-2}$$

$$= \frac{1}{4}\left[1 + -2\left(-\frac{5}{2}x\right) + \frac{-2.-3}{1.2}\left(-\frac{5}{2}x\right)^2 + \frac{-2.-3.-4}{1.2.3}\left(-\frac{5}{2}x\right)^3 + \ldots\right]$$

$$= \frac{1}{4}\left[1 + 5x + \frac{75}{4}x^2 + \frac{125}{2}x^3 + \ldots\right]$$

$$= \frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2 + \frac{125}{8}x^3 + \ldots.$$



N23562A0220

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**1.** $$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f($x$), in ascending powers of $x$, as far as the term in $x^3$.

Give each coefficient as a simplified fraction.

**(5)**

$$(3+2x)^{-3} = \left(3\left(1+\tfrac{2}{3}x\right)\right)^{-3} = 3^{-3}\left(1+\tfrac{2}{3}x\right)^{-3}$$

$$= \frac{1}{27}\left[1 + -3\left(\frac{2}{3}x\right) + \frac{-3.-4}{1.\,2}\left(\frac{2}{3}x\right)^2 + \frac{-3.-4.-5}{1.\,2.\,3}\left(\frac{2}{3}x\right)^3 + \ldots\right]$$

$$= \frac{1}{27}\left[1 - 2x + \frac{8}{3}x^2 - \frac{80}{27}x^3 + \ldots\right]$$

$$= \frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80x^3}{729} + \ldots$$



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**2.** (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of $x$, up to and including the term in $x^3$, giving each term as a simplified fraction.

**(5)**

(b) Use your expansion, with a suitable value of $x$, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

**(2)**

a) $(8-3x)^{\frac{1}{3}} = \left(8\left(1-\frac{3x}{8}\right)\right)^{\frac{1}{3}} = 2\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$

$= 2\left[1 + \frac{1}{3}\left(-\frac{3x}{8}\right) + \frac{\frac{1}{3}\cdot-\frac{2}{3}}{1.2}\left(-\frac{3x}{8}\right)^2 + \frac{\frac{1}{3}\cdot-\frac{2}{3}\cdot-\frac{5}{3}}{1.2.3}\left(-\frac{3x}{8}\right)^3 + \dots\right]$

$= 2\left[1 - \frac{x}{8} - \frac{x^2}{64} - \frac{5x^3}{1536} - \dots\right]$

$= 2 - \frac{x}{4} - \frac{x^2}{32} - \frac{5x^3}{768} - \dots$

b) $x = 0.1$

$\sqrt[3]{7.7} \approx 2 - \frac{0.1}{4} - \frac{0.1^2}{32} - \frac{5 \times 0.1^3}{768}$

$= 1.97468099$

$= 1.9746810$ to 7 d.p.



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**5.** (a) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of $x$ up to and including the term in $x^2$. Simplify each term.

**(5)**

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of $x$.

**(4)**

a) $(4-3x)^{-\frac{1}{2}} = \left(4\left(1-\frac{3x}{4}\right)\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left[1 + -\frac{1}{2}\left(-\frac{3x}{4}\right) + \frac{-\frac{1}{2}\cdot-\frac{3}{2}}{1\cdot 2}\left(-\frac{3x}{4}\right)^2 + \dots\right]$

$= \frac{1}{2}\left[1 + \frac{3x}{8} + \frac{27x^2}{128} + \dots\right]$

$= \frac{1}{2} + \frac{3x}{16} + \frac{27x^2}{256} + \dots$

b) $\dfrac{(x+8)}{\sqrt{4-3x}} \approx (x+8)\left(\frac{1}{2} + \frac{3x}{16} + \frac{27x^2}{256}\right)$

$\approx \frac{1}{2}x + 4 + \frac{3x^2}{16} + \frac{3x}{2} + \frac{27x^2}{32}$

$= 4 + 2x + \frac{33x^2}{32}$



H 3 0 4 2 7 A 0 1 4 2 8

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**1.** $$\mathrm{f}(x) = \frac{1}{\sqrt{(4+x)}}, \qquad |x| < 4$$

Find the binomial expansion of $\mathrm{f}(x)$ in ascending powers of $x$, up to and including the term in $x^3$. Give each coefficient as a simplified fraction.

**(6)**

$$(4+x)^{-\frac{1}{2}} = \left(4\left(1+\frac{x}{4}\right)\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\left[1 + -\frac{1}{2}\left(\frac{x}{4}\right) + \frac{-\frac{1}{2}\cdot-\frac{3}{2}}{1.2}\left(\frac{x}{4}\right)^2 + \frac{-\frac{1}{2}\cdot-\frac{3}{2}\cdot-\frac{5}{2}}{1.2.3}\left(\frac{x}{4}\right)^3 + \ldots\right]$$

$$= \frac{1}{2}\left[1 - \frac{x}{8} + \frac{x^2}{128} - \frac{5x^3}{1024} + \ldots\right]$$

$$= \frac{1}{2} - \frac{x}{16} + \frac{x^2}{256} - \frac{5x^3}{2048} + \ldots$$

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**1.** (a) Find the binomial expansion of

$$\sqrt{(1-8x)}, \quad |x| < \frac{1}{8},$$

in ascending powers of $x$ up to and including the term in $x^3$, simplifying each term.

**(4)**

(b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

**(2)**

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

**(3)**

a) $(1-8x)^{\frac{1}{2}}$

$= \left[1 + \frac{1}{2}(-8x) + \frac{\frac{1}{2}\cdot -\frac{1}{2}}{1\cdot 2}(-8x)^2 + \frac{\frac{1}{2}\cdot -\frac{1}{2}\cdot -\frac{3}{2}}{1\cdot 2\cdot 3}(-8x)^3 + \ldots\right]$

$= 1 - 4x - 8x^2 - 32x^3 - \ldots$

b) $x = \frac{1}{100}$, $\sqrt{1-8x} = \sqrt{0.92} = \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5}$

c) $\frac{\sqrt{23}}{5} \approx 1 - 0.04 - 0.0008 - 0.000032 = 0.959168$

$\sqrt{23} \approx 0.959168 \times 5$

$= 4.79584$