Discrete Random Variables

Let X be a discrete random variable

Expected value of X written as $E(X)$

is given by

$$E(X) = \sum_i x_i P(X = x_i)$$

Example A dice

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \frac{1}{6}\times 1 + \frac{1}{6}\times 2 + \frac{1}{6}\times 3 + \frac{1}{6}\times 4 + \frac{1}{6}\times 5 + \frac{1}{6}\times 6$$

$$= \frac{1}{6}\left[1+2+3+4+5+6\right]$$

$$= \frac{21}{6} = 3.5$$

$$Var(X) = E(X^2) - (E(X))^2$$

Variance is the Expectation of the Squares minus the Square of the Expectation

---

Ex Die again

x^2	1	4	9	16	25	36
x	1	2	3	4	5	6
p	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(x^2) = \frac{1}{6}\times 1 + \frac{1}{6}\times 4 + \frac{1}{6}\times 9 + \frac{1}{6}\times 16 + \frac{1}{6}\times 25 + \frac{1}{6}\times 36$$

$$= \frac{1}{6}\left[1 + 4 + 9 + 16 + 25 + 36\right]$$

$$= \frac{91}{6}$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$= \frac{91}{6} - 3.5^2$$

$$= \frac{35}{12}$$

---

4 The number, X, of children per family in a certain city is modelled by the probability distribution $P(X = r) = k(6 - r)(1 + r)$ for $r = 0, 1, 2, 3, 4$.

(i) Copy and complete the following table and hence show that the value of k is $\frac{1}{50}$. [3]

r	0	1	2	3	4
$P(X = r)$	$6k$	$10k$	$12k$	$12k$	$10k$

(ii) Calculate $E(X)$. [2]

(iii) Hence write down the probability that a randomly selected family in this city has more than the mean number of children. [1]

i) $6k + 10k + 12k + 12k + 10k = 1$

$50k = 1 \Rightarrow k = \frac{1}{50}$

ii) $E(X) = \sum x P(X=x)$

$= \frac{1}{50}\left[6\times 0 + 10\times 1 + 12\times 2 + 12\times 3 + 10\times 4\right]$

$= \frac{110}{50} = \frac{11}{5} = 2.2$

---

iii) $P(3 \text{ or } 4) = \frac{12}{50} + \frac{10}{50} = \frac{22}{50} = 0.44$

---

3 Jeremy is a computing consultant who sometimes works at home. The number, X, of days that Jeremy works at home in any given week is modelled by the probability distribution

$$P(X = r) = \frac{1}{40}r(r+1) \quad \text{for } r = 1, 2, 3, 4.$$

(i) Verify that $P(X = 4) = \frac{1}{2}$. [1]

(ii) Calculate $E(X)$ and $\mathrm{Var}(X)$. [5]

(iii) Jeremy works for 45 weeks each year. Find the expected number of weeks during which he works at home for exactly 2 days. [2]

i) $P(X=4) = \frac{1}{40}\times 4\times 5 = \frac{20}{40} = \frac{1}{2}$

ii)

r^2	1	4	9	16
r	1	2	3	4
$P(X=r)$	$\frac{2}{40}$	$\frac{6}{40}$	$\frac{12}{40}$	$\frac{20}{40}$

$E(X) = \frac{1}{40}\left[2\times 1 + 6\times 2 + 12\times 3 + 20\times 4\right]$

$$E(X) = \frac{130}{40} = \frac{13}{4} = 3.25$$

$$E(X^2) = \frac{1}{40}\left[2\times1 + 6\times4 + 12\times9 + 20\times16\right]$$

$$= \frac{454}{40}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$= \frac{454}{40} - 3.25^2$$

$$= \frac{63}{80}$$

iii) $45 \times \frac{6}{40} = \frac{27}{4}$ or 6.75

2 Four letters are taken out of their envelopes for signing. Unfortunately they are replaced randomly, one in each envelope.

The probability distribution for the number of letters, X, which are now in the correct envelope is given in the following table.

r^2	0	1	4	9	16
r	0	1	2	3	4
$P(X=r)$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	0	$\frac{1}{24}$

(i) Explain why the case $X = 3$ is impossible. [1]

(ii) Explain why $P(X = 4) = \frac{1}{24}$. [2]

(iii) Calculate $E(X)$ and $\text{Var}(X)$. [5]

i) If 3 are correct, the only envelope left for the 4^{th} is the correct one

ii) $\boxed{4 \mid 3 \mid 2 \mid 1}$ $\quad 4!$ ways $= 24$

only one correct so $P(X=4) = \frac{1}{24}$

iii)

$$E(X) = \frac{3}{8} \times 0 + \frac{1}{3} \times 1 + \frac{1}{4} \times 2 + \frac{1}{24} \times 4$$

$$E(X) = 1$$

$$E(X^2) = \frac{3}{8} \times 0 + \frac{1}{3} \times 1 + \frac{1}{4} \times 4 + \frac{1}{24} \times 16$$

$$E(X^2) = 2$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 2 - 1^2$$

$$Var(X) = 1$$

3 The score, X, obtained on a given throw of a biased, four-faced die is given by the probability distribution

$$P(X = r) = kr(8 - r) \text{ for } r = 1, 2, 3, 4.$$

(i) Show that $k = \frac{1}{50}$. [2]

(ii) Calculate $E(X)$ and $Var(X)$. [5]

6 In a phone-in competition run by a local radio station, listeners are given the names of 7 local personalities and are told that 4 of them are in the studio. Competitors phone in and guess which 4 are in the studio.

(i) Show that the probability that a randomly selected competitor guesses all 4 correctly is $\frac{1}{35}$. [2]

Let X represent the number of correct guesses made by a randomly selected competitor. The probability distribution of X is shown in the table.

r	0	1	2	3	4
$P(X = r)$	0	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

(ii) Find the expectation and variance of X. [5]