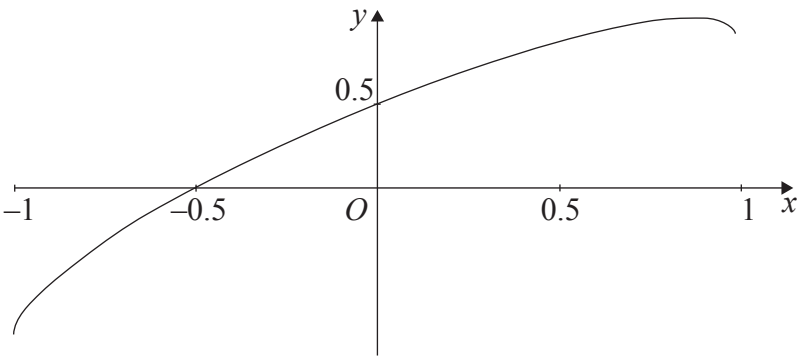
Leave blank

4. **Figure 2**



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

a) $\frac{dx}{dt} = \cos t \qquad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{\cos\left(t + \frac{\pi}{6}\right)}{\cos t}$

when $t = \frac{\pi}{6}$ $\qquad \frac{dy}{dx} = \frac{\cos\frac{\pi}{3}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

when $t = \frac{\pi}{6}$

$x = \sin\frac{\pi}{6} = \frac{1}{2}$

$y = \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Tangent $\qquad y - y_1 = m(x - x_1)$

N 2 3 5 6 3 A 0 8 2 0

Leave blank

Question 4 continued

$$y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$

$$\sqrt{3}y - \frac{3}{2} = x - \frac{1}{2}$$

$$0 = x - \sqrt{3}y + 1$$

b) $x = \sin t \qquad y = \sin\left(t + \frac{\pi}{6}\right)$

$$y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$$

$$y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$$

$$y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{1 - \sin^2 t}$$

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1 - x^2}$$

Q4

(Total 9 marks)