UPPER AND LOWER BOUNDS

[ESTIMATED TIME: 70 minutes]

GCSE

(+ IGCSE) EXAM QUESTION PRACTICE

1. **[2 marks]**

The length of a fence is 137 metres, correct to the nearest metre.

Write down

(i) the lower bound for the length of the fence,

136.5 metres

(ii) the upper bound for the length of the fence.

137.5 metres

2. **[5 marks]**

Zoe's weight is 62 kg, correct to the nearest kilogram.
Anu's weight is 85 kg, correct to the nearest kilogram.

(a) Write down the upper bound for the weight of Zoe.

62.5 kg

(1)

(b) Write down the lower bound for the weight of Anu.

84.5 kg

(1)

(c) Work out the upper bound for the difference between Zoe's weight and Anu's weight.

85.5 − 61.5

24 kg

(3)

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3. **[2 marks]**

$y = 1.8$ correct to 1 decimal place.

Calculate the lower bound for the value of $4y + 1$

$4(1.75) + 1 = 8$

8

4. **[3 marks]**

The volume of oil in a tank is 1000 litres, correct to the nearest 10 litres.
The oil is poured into tins of volume 2.5 litres, correct to one decimal place.

Calculate the upper bound of the number of tins which will be required.

$\frac{1005}{2.45} = 410.2$

411

5. **[7 marks]**

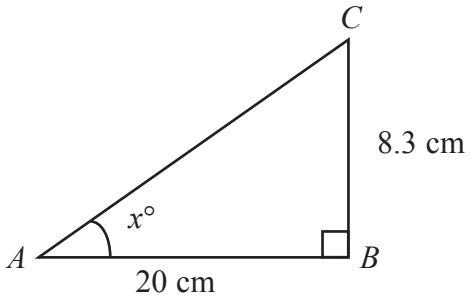


Diagram **NOT** accurately drawn

Triangle *ABC* is right-angled at *B*.
AB = 20 cm, correct to 1 significant figure.
BC = 8.3 cm, correct to 2 significant figures.

(a) Write down the lower bound for the length of

(i) *AB*,

15 cm

(ii) *BC*.

8.25 cm

(2)

(b) Calculate the lower bound for the area of triangle *ABC*.

$\frac{1}{2} \times 15 \times 8.25$

61.875 cm^2

(2)

(c) Calculate the lower bound for the value of tan *x*°.

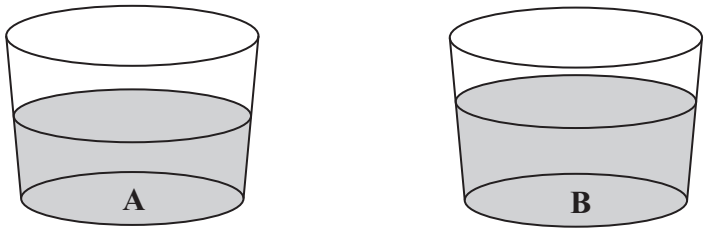
LB for $\tan x = \frac{8.25}{25} = 0.33$

smallest / largest

0.33

(3)

6. **[2 marks]**



Glass **A** contains 122 millilitres of water, correct to the nearest millilitre.
Glass **B** contains 168 millilitres of water, correct to the nearest millilitre.

Calculate the upper bound of the difference, in millilitres, between the volume of water in glass **A** and the volume of water in glass **B**.

$168.5 - 121.5 = 47$ ml

47 millilitres

7. **[3 marks]**

An athlete runs 400 metres, correct to the nearest metre.
The athlete takes 50.2 seconds, correct to the nearest 0.1 of a second.

Work out the upper bound of the athlete's average speed.
Give your answer correct to 3 significant figures.

$S = \frac{D}{T}$

upper bound for $S = \frac{400.5}{50.15}$

$= 7.986$ m/s

7.99 m/s

8. **[3 marks]**

$$y = \frac{2a}{b - c}$$

$a = 42$ correct to 2 significant figures.
$b = 24$ correct to 2 significant figures.
$c = 14$ correct to 2 significant figures.

Work out the lower bound for the value of y.
Give your answer correct to 2 significant figures.
Show your working clearly.

lower bound for $y = \frac{\text{small}}{\text{large}}$

$$y = \frac{2 \times 41.5}{(24.5 - 13.5)} = \frac{83}{11}$$

$= 7.545$

$= 7.5$ to 2 s.f

7.5

9. **[3 marks]**

There are 1300 sheets of paper, correct to the nearest 100 sheets, in a pile.
Each sheet is of equal thickness.
The height of the pile is 160 mm, correct to the nearest 10 mm.

Calculate the upper bound, in millimetres, for the thickness of one sheet of paper.

$$UB = \frac{\text{large}}{\text{small}} = \frac{165}{1250} = \frac{33}{250} = 0.132 \text{ mm}$$

0.132 mm

10. **[4 marks]**

Some cases have to be lifted by a crane.
Each case has a mass of 68 kg, correct to 2 significant figures.

(a) Write down the upper bound of the mass of a case.

68.5 kg
(1)

A crane can lift safely a load of 1200 kg, correct to 2 significant figures.

(b) Find the greatest number of cases that the crane can lift safely in one load.

$\frac{1150}{68.5} = 16.78$

so 16 cases

16
(3)

11. **[4 marks]**

The length of a side of a square is 6.81 cm, correct to 3 significant figures.

(a) Work out the lower bound for the perimeter of the square.

LB = 6.805 × 4

27.22 cm
(2)

(b) Give the perimeter of the square to an appropriate degree of accuracy.
You must show working to explain how you obtained your answer.

UB = 6.815 × 4
= 27.26 cm

LB and UB agree to 2 s.f.

27 cm
(2)

12. **[6 marks]**

(a) Make r the subject of the formula $A = \pi r^2$, where r is positive.

$$\frac{A}{\pi} = r^2$$

$$r = \sqrt{\frac{A}{\pi}}$$

$r =$ $\sqrt{\frac{A}{\pi}}$

(2)

The area of a circle is 14 cm^2, correct to 2 significant figures.

(b) (i) Work out the lower bound for the radius of the circle.
Write down all the figures on your calculator display.

LB $r = \sqrt{\frac{13.5}{\pi}}$

2.072964897 cm

(ii) Give the radius of the circle to an appropriate degree of accuracy.
You must show working to explain how you obtained your answer.

UB $r = \sqrt{\frac{14.5}{\pi}} = 2.148369928$

LB and UB agree to 2 s.f.

2.1 cm

(4)

13. **[4 marks]**

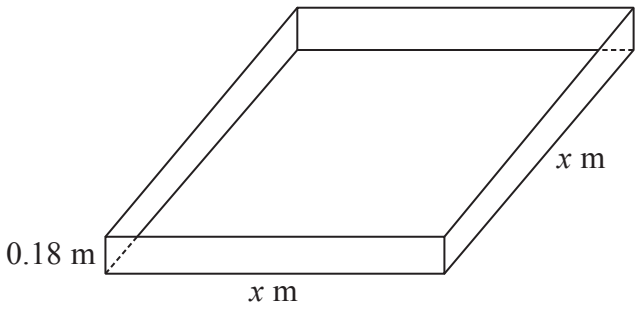


Diagram **NOT** accurately drawn

Trena wants to build a sandpit in the shape of a cuboid.
The volume of sand in the sandpit will be 1.0 m^3, correct to 1 decimal place.
The depth of sand in the sandpit will be 0.18 metres, correct to 2 decimal places.
The sandpit will have a square base with sides of length x metres.

Find the upper bound for x
Give your answer correct to 3 significant figures.

UB for $x = \sqrt{\frac{1.05}{0.175}} = 2.45$ m

to 3 s.f.

upper bound = 2.45 m

14. **[5 marks]**

(a) Correct to the nearest millimetre, the length of a side of a regular hexagon is 3.6 cm.

Calculate the upper bound for the perimeter of the regular hexagon.

3.65×6

21.9 cm

(2)

(b) Correct to 1 significant figure, the area of a rectangle is 80 cm^2
Correct to 2 significant figures, the length of the rectangle is 12 cm.

Calculate the lower bound for the width of the rectangle.
Show your working clearly.

LB for width $= \frac{\text{small area}}{\text{large length}} = \frac{75}{12.5} = 6$

6 cm

(3)

15. **[3 marks]**

Correct to 2 significant figures, the area of a square is 230 cm^2.

Calculate the lower bound for the perimeter of the square.

LB Length $= \sqrt{225} = 15$ cm

LB Perim $= 15 \times 4 = 60$ cm

60 cm

16. **[4 marks]**

Correct to 2 decimal places, the volume of a solid cube is 42.88 cm^3

Calculate the lower bound for the surface area of the cube.

LB side $= \sqrt[3]{42.875} = 3.5$

LB surface area $= 6 \times 3.5^2$

$= 73.5\ cm^2$

..... cm^2

17. **[3 marks]**

Correct to 2 significant figures, $a = 58$, $b = 28$ and $c = 18$

Calculate the upper bound for the value of $\dfrac{a}{b-c}$

Show your working clearly.

$$\frac{58.5}{27.5 - 18.5}$$

$= 6.5$

.....

18. **[5 marks]**

A metal cube has sides of length 4.5 cm, correct to the nearest 0.5 cm.

The cube is melted down and the metal is used to make small spheres.
Each sphere has a radius of 3 mm, correct to the nearest millimetre.

Work out the greatest number of spheres that could be made from the metal.
Show your working clearly.

Largest cube $47.5^3 = 107172$ mm^3

Smallest sphere $= \frac{4}{3}\pi \times 2.5^3$ mm^3

$= 65.4498$ mm^3

$\frac{107172}{65.4498} = 1637.47$

so maximum of 1637 spheres

.....

19. **[5 marks]**

Correct to 1 significant figure, $x = 7$ and $y = 9$

(a) Calculate the lower bound for the value of xy

6.5×8.5

55.25

(2)

(b) Calculate the upper bound for the value of $\frac{x}{y}$

$\frac{7.5}{8.5}$

0.882 to 3 s.f.

(3)

20. **[4 marks]**

In a race, Paula runs 25 laps of a track.
Each lap of the track is 400 m, correct to the nearest metre.
Paula's average speed is 5.0 m/s, correct to one decimal place.

Calculate the upper bound for the time that Paula takes to run the race.
Give your answer in minutes and seconds, correct to the nearest second.

UB for time $= \frac{25 \times 400.5}{4.95}$

$= 2022.73$ s

$= 2023$ s

$=$ 33 min 43 seconds