

- (P) 4 The first three terms in the binomial expansion of  $\frac{1}{\sqrt{a+bx}}$  are  $3 + \frac{1}{3}x + \frac{1}{18}x^2 + \dots$

a Find the values of the constants  $a$  and  $b$ .

b Find the coefficient of the  $x^3$  term in the expansion.

$$\begin{aligned} \frac{1}{\sqrt{a+bx}} &= \frac{1}{\sqrt{a(1+\frac{b}{a}x)}} = \frac{1}{\sqrt{a}(1+\frac{b}{a}x)^{\frac{1}{2}}} = \frac{1}{\sqrt{a}} \left(1 + \frac{b}{a}x\right)^{-\frac{1}{2}} \\ &\approx \frac{1}{\sqrt{a}} \left(1 + -\frac{1}{2}\left(\frac{b}{a}x\right) + \frac{-\frac{1}{2} \cdot -\frac{3}{2}}{1 \cdot 2} \left(\frac{b}{a}x\right)^2\right) + \frac{-\frac{1}{2} \cdot \frac{1}{2} \cdot -\frac{5}{2}}{1 \cdot 2 \cdot 3} \left(\frac{b}{a}x\right)^3 \\ &\quad \frac{1}{\sqrt{a}} \left(1 - \frac{b}{2a}x + \frac{3b^2}{8a^2}x^2\right) \\ \frac{1}{\sqrt{a}} &- \frac{b}{2a\sqrt{a}}x + \frac{3b^2}{8a^2\sqrt{a}}x^2 \\ \Rightarrow \frac{1}{\sqrt{a}} &= 3 \\ 1 &= 3\sqrt{a} \Rightarrow \sqrt{a} = \frac{1}{3} \Rightarrow a = \frac{1}{9} \\ -\frac{b}{2a\sqrt{a}} &= \frac{1}{3} \\ \Rightarrow -\frac{b}{2 \times \frac{1}{9} \times \frac{1}{3}} &= \frac{1}{3} \\ \Rightarrow -\frac{b}{\frac{2}{27}} &= \frac{1}{3} \\ -\frac{27b}{2} &= \frac{1}{3} \\ -b &= \frac{2}{3 \times 27} \qquad b = -\frac{2}{81} \end{aligned}$$

$$\begin{aligned}
 \text{Coeff of } x^3 &= -\frac{5}{16} \frac{b^3}{a^3} \\
 &= -\frac{5}{16} \times \frac{\left(-\frac{z}{8}\right)^3}{\left(\frac{1}{q}\right)^3} \times \frac{1}{\sqrt[3]{\frac{1}{q}}} \\
 &= + \frac{5}{16} \times \frac{8}{(80)^3} \times 3 \times 729 \\
 &= \frac{5}{486}
 \end{aligned}$$

(P) 5 a Express  $\frac{2x^2 + 7x - 6}{(x+5)(x-4)}$  in partial fractions.

**Hint** First divide the numerator by the denominator.

b Hence, or otherwise, expand  $\frac{2x^2 + 7x - 6}{(x+5)(x-4)}$  in ascending powers of  $x$  as far as the term in  $x^2$ .

c State the set of values of  $x$  for which the expansion is valid.

$$x^2 + x - 20 \overline{)2x^2 + 7x - 6}$$

$$\begin{array}{r} 2x^2 + 2x - 40 \\ \hline + 5x + 34 \end{array}$$

$$\frac{2x^2 + 7x - 6}{(x+5)(x-4)} = 2 + \frac{5x + 34}{(x+5)(x-4)}$$

Aside

$$\frac{5x + 34}{(x+5)(x-4)} = \frac{A}{x+5} + \frac{B}{x-4}$$

$$5x + 34 = A(x-4) + B(x+5)$$

$$x=4 \quad 54 = 9B \Rightarrow B = 6$$

$$x=-5 \quad 9 = -9A \Rightarrow A = -1$$

$$= 2 - \frac{1}{x+5} + \frac{6}{x-4}$$

$$= 2 - (x+5)^{-1} + 6(x-4)^{-1}$$

$$= 2 - (5(1+\frac{x}{5}))^{-1} - 6(4-x)^{-1}$$

$$\begin{aligned}
&= 2 - \frac{1}{5} \underbrace{\left(1 + \frac{x}{5}\right)^{-1}}_{1 + -1\left(\frac{x}{5}\right) + \frac{-1 \cdot -2}{1 \cdot 2}\left(\frac{x}{5}\right)^2} - \frac{6}{4} \underbrace{\left(1 - \frac{x}{4}\right)^{-1}}_{1 + -1\left(-\frac{x}{4}\right) + \frac{-1 \cdot -2}{1 \cdot 2}\left(-\frac{x}{4}\right)^2} \\
&= 2 - \frac{1}{5} \left(1 - \frac{x}{5} + \frac{x^2}{25}\right) - \frac{3}{2} \left(1 + \frac{x}{4} + \frac{x^2}{16}\right) \\
&= 2 - \frac{1}{5} + \frac{x}{25} - \frac{x^2}{125} - \frac{3}{2} - \frac{3x}{8} - \frac{3x^2}{32} \\
&= \frac{3}{10} - \frac{67}{200}x - \frac{407}{4000}x^2
\end{aligned}$$

$$|\frac{x}{5}| < 1 \Rightarrow |x| < 5$$

$$|\frac{x}{4}| < 1 \Rightarrow |x| < 4$$

Overall  $|x| < 4$

$$-4 < x < 4$$


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